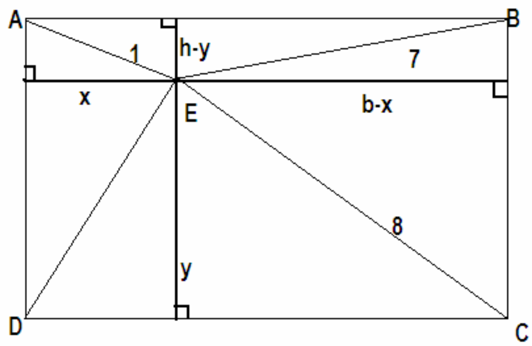
1. B
2. D
3. B
4. A
5. D
6. B
7. C
8. E (4/5)
9. A
10. E (I, II, and III)
11. A
12. D
13. C
14. C
15. C
16. A
17. C
18. B
19. C
20. C
21. B
22. B
23. A
24. D
25. B
26. A
27. E (1800°)
28. C
29. A
30. D

1. $90 - A = \frac{1}{3}(180 - A)$; $45 = A$ **B**
2. The Fibonacci sequence has the property that each term is the sum of the previous two terms. A triangle having a side that is exactly the sum of the other two won't be a triangle—it will be a straight line => degenerate. **D**
3. $3^2 + 4^2 > 4^2 \rightarrow$ acute, isosceles **B**
4. (side of B) / (side of A) = 2 : 1, so (area of B) / (area of A) = 4 : 1; similarly, no pun intended, (area of C) / (area of B) = 4 : 1. Thus desired ratio is 16 : 1 **A**
5. $7^2 + 12^2 \neq 13^2$ **D**
6. Triangles ACF and DCE are similar, ratio of sides is 5 : 1 and so ratio of areas is 25 : 1. Area of ACF is 5*2/2 = 5, so answer is 1 / 5. **B**
7. Angles must be 30°, 30° and 120°. Draw altitude and use 30°-60°-90° properties to find altitude. **C**
8. Call the sides a, b, and c opposite angles CAB, ABC, and BCA, respectively. Calculate area of triangle using each side as a base: $\frac{3a}{2} = \frac{4b}{2} = \frac{5c}{2}$, so a / b = 4/ 3. Due to the Law of Sines, $\frac{\sin \angle ABC}{b} = \frac{\sin \angle CAB}{a}$. Solve to get the answer of 4 / 5. **E**
9. $\sqrt{6^2 + 8^2} + \sqrt{(9-6)^2 + 4^2}$ = 15 m **A**
10. Statements I, II, and III are incorrect. Only statement IV is correct. (Note: not adjacent because no ray in common) **E**
11. Referring to angles CAB, ABC and BCA as angles A, B and C, respectively, we have that A + B + C = 180° and are given A + C = 2B, subtracting, we get 3B = 180° and B = 60°, so x = 50° **A**
12. Sum of bisected angles before bisection: 90°; after bisection: 45°. Obtuse angle of intersection is then $180 - 45 = 135^\circ$. **D**
13. Area is ab / 2. Square both sides of the Pythagorean Thm and subtract $a^4 + b^4$ from both sides, then divide by 8 to get the desired expression. It's equal to the square of the triangle's area, or 900. **C**
14. By similarity, area of triangle EPQ is 6. Area of trapezoid is 24 – 6 = 18, set this equal to $\frac{s^2}{2}$, where s is a leg of the 45-45-90 degree triangle. Solve to get s = 6, perimeter is then $12 + 6\sqrt{2}$. **C**
15. $\frac{16}{16+38} = \frac{6}{x}$, x = $\frac{81}{4}$ **C**
16. Inradius = (area of triangle) / (semiperimeter of triangle) = $\frac{\sqrt{2}\cdot\sqrt{2}}{2} \div \left(1+\sqrt{2}\right) = \sqrt{2} - 1$ **A**
17. Goat can graze only in a circular area in the corner of the triangle with the 60° angle (1/6 the area of a circle of radius 1), because the length of the rope is too short to reach the pasture by going around the fence w/ length $6\sqrt{3}$ m. **C**
18. One side length (from (0,0) to (b,0)) has length b. Use the distance formula to set b equal to the distances between other vertices. Solve system of equations to find $a = 3, b = 6,$ or $a = -3, b = -6, \therefore ab = 18$. **B**
19. Side of such a triangle is equal to length of diagonal of a face of the cube, which is $s = 7\sqrt{2}$. $s^2 \frac{\sqrt{3}}{4} = \frac{49\sqrt{3}}{2}$. **C**

20. If CB is perpendicular to AB, the triangle is a right triangle with x = $90^\circ - \theta$. For angles x smaller than this value, the triangle will be obtuse because then angle ABC must exceed 90°. So $90^\circ - \theta$ is the minimum value of x, which also has an obvious maximum value of 90°. The triangle will therefore be acute when x is between $90^\circ - \theta$ and 90°. Probability = $\frac{90°-(90°-\theta)}{180^\circ-\theta} \Rightarrow \frac{\theta}{180^\circ - \theta}$.**C**
21. Angle CGE = BGF (vertical angles), and angles AGF, BGF and BGD form a straight angle (180°). **B**
22. Using the Pythagorean Thm, represent the (squares of) the lengths 1, 7, 8, and ED as the sum of the squares of two of the following: x, y, b-x, or h-y. For example, $7^2 = (h-y)^2 + (b-x)^2$. Notice that $1^2 + 8^2 = (ED)^2 + 7^2$ is identically true. Thus ED = 4. **B**



Since AB = CD, use the Law of Cosines for each of the triangles AEB and DEC (with AB or CD as the unknown side), and set the expressions equal. We're given cos (AEB) = -cos(DEC), so plug this in and solve for cos (AEB), which is -5/13, then plug back in to find AB = root(720/13). Angles AED and BEC must sum to 180° as well, so doing the same for triangles AED and BEC, we get cos AED = -4/5, and AD = root(117/5). The area is AB*AD = 36.

23. Drawing a line segment from point A to C creates two 30°-60°-90° triangles with shortest leg 4 (the radius). Area of quadrilateral is then $4 \cdot 4\sqrt{3} = 16\sqrt{3}$ **A**
24. Call sides $n-x, n,$ and $n+x$ (arithmetic). Then $\frac{n+x}{n} = \frac{n}{n-x}$ (geometric). Cross-multiply to find x = 0, thus triangle is equilateral. Inradius of length 2 is given by area / semiperimeter = $\left(\frac{s^2\sqrt{3}}{4}\right) \div \left(\frac{3s}{2}\right)$, s = $4\sqrt{3}$.
Circumradius = (product of sides) /(4·Area) = 4, so circumcircle's area is 16π. **D**
25. The segments parallel to EF have lengths 40j/6 for j = 1….5, and the height of each trapezoid is 30/6. Using the formula for the area of a trapezoid, the shaded area is:
(30/6)/2 * ((40(1)/6 + 40(2)/6) + (40(3)/6 + (40(4)/6)) + (40(5)/6 + 40))
= 15/6 (21)(40)/6 = 350; so fraction of total area is 350/600 = 7/12 **B**
26. After 12 clockwise turns, you'll have turned full circle to make a dodecagon. **A**
27. sum of interior angles = (12-2)(180)=1800° **E**
28. $s^2 \frac{\sqrt{3}}{4} = 25\sqrt{3}$, so s = 10. Draw altitude to base, which will intersect the base at the centroid, 2/3 the length of a side from each vertex. Use this to find altitude and then volume. **C**
29. Area of triangle is 6, which must be equal to bh/2 = 5h / 2. AE is then 12/5. Median to hypotenuse of a right triangle is half the hypotenuse, so AD is 5/2. By the Pythagorean Thm, DE is 7/10. So answer is (7/10)/(5/2) = 7/25 **A**
30. Side $s = \frac{10}{\sqrt{3}}$. One type of diagonal, that bisects the hexagon, has length 2s, while the other type of diagonal has length $s\sqrt{3}$. There can only be 3 diagonals of the first type, so there are 6(6-3)/2 – 3 = 6 diagonals of the second type. This makes for a total length of $6s + 6s\sqrt{3} = 60 + 20\sqrt{3}$. **D**