1. The complement of $\angle A$ is equal to one-third of the supplement of $\angle A$. Find $m \angle A$.

A. 30°  B. 45°  C. 60°  D. 75°  E. NOTA

2. A triangle having which of the following properties MUST be degenerate?

A. The sum of any two sides is greater than the third side  
B. The triangle has side lengths 1, 1, and $\sqrt{3}$.  
C. The product of the smallest two sides is less than the square of the third side.  
D. The side lengths are any 3 consecutive terms of the Fibonacci sequence (this sequence begins 1, 1, 2, 3, 5, …)  
E. NOTA

3. A triangle has sides of length 3, 4, and 4. It is:

A. obtuse, scalene  B. acute, isosceles  C. acute, scalene  D. obtuse, isosceles  E. NOTA

4. The longest side of triangle A is 8. Triangle A is similar to triangle B, whose longest side 16. Triangle B is similar to triangle C, whose longest side 32. What is the ratio of the area of triangle C to the area of triangle A?

A. 16 : 1  B. 25 : 1  C. 9 : 1  D. 4 : 1  E. NOTA

5. Which is NOT a right triangle? A triangle with side lengths:

A. 3, 4, 5  B. $\frac{5}{2}$, 6, $\frac{13}{2}$  C. 9, 40, 41  D. 7, 12, 13  E. NOTA

6. Find the area of the shaded region if AC = 5, CD = 1, and altitude FB = 2:

A. 1  B. $\frac{1}{5}$  C. 2  D. 5  E. NOTA

7. The sum of the measures of the two base angles in an isosceles triangle is 60°. If the base of the triangle has a length of 10, find the length of the altitude to the base.

A. $\sqrt{3}$  B. 6  C. $\frac{5\sqrt{3}}{3}$  D. 10  E. NOTA
8. In $\triangle ABC$, the altitude dropped from vertex A has length 3, the altitude dropped from vertex B has length 4, and the altitude dropped from vertex C has length 5. If $\sin \angle ABC = \frac{3}{5}$, what is $\sin \angle CAB$?

A. $\frac{3}{4}$  
B. $\frac{2}{5}$  
C. 1  
D. $\frac{3}{5}$  
E. NOTA

9. Wires are stretched from the top of each of two poles (of heights 8 m and 4 m) to a single point on the ground lying on the straight line between their positions. If the poles are 9 m apart, and the point on the ground where the wires touch is 6 m from the 8 m high pole, what is the total length of wire used?

A. 15 m  
B. 17 m  
C. 20 m  
D. 22 m  
E. 25 m

10. Which of the following statements is (are) NOT correct regarding quadrilateral ABCD?

I. $\angle ABE$ and $\angle DBC$ are adjacent angles.  
II. $\angle ABC$ is trisected.  
III. $\triangle ABE \cong \triangle BDE \cong \triangle DBC$  
IV. $\triangle ABE \sim \triangle BDE \sim \triangle DBC$

A. II and III  
B. II  
C. I and II  
D. III and IV  
E. NOTA

11. Given the angle measures below and that $m\angle BCA + m\angle CAB = 2 \cdot m\angle ABC$, find $x$:

A. 50  
B. 40  
C. 75  
D. 72  
E. NOTA

12. Consider the angle bisectors of the acute angles in a right triangle. Find the measure of the obtuse angle of their intersection.

A. 109°  
B. 120°  
C. 150°  
D. 135°  
E. NOTA
13. The well-known Pythagorean Theorem states that $a^2 + b^2 = c^2$ in a right triangle with legs $a$ and $b$ and hypotenuse $c$. Find $\frac{\sqrt{c^4 - (a^4 + b^4)}}{8}$ if the area of such a triangle equals 30.

A. 300  B. 600  C. 900  D. 450  E. NOTA

14. In $\triangle BED$, points P and Q are the midpoints of sides BE and ED, respectively. If the area of $\triangle BED$ is 24, what must be the perimeter of a $45^\circ$-$45^\circ$-$90^\circ$ triangle if it is to have the same area as quadrilateral PQDB?

A. $2\sqrt{2}$  B. $2 + 2\sqrt{2}$  C. $12 + 6\sqrt{2}$  D. $8 + 8\sqrt{2}$  E. NOTA

15. One afternoon, the shadow of a man who is 6 ft tall is contained completely within the shadow of a building. He stands 38 ft away from the building and is positioned in such a way that the tip of his 16 ft shadow exactly coincides with the tip of the building’s shadow. How tall is the building (in feet)?

A. $\frac{57}{4}$  B. $\frac{114}{11}$  C. $\frac{81}{4}$  D. $\frac{81}{11}$  E. NOTA

16. What is the radius of a circle inscribed in a $45^\circ$-$45^\circ$-$90^\circ$ triangle with hypotenuse length 2?

A. $\sqrt{2} - 1$  B. $\frac{\sqrt{2} + 1}{2}$  C. $\frac{1}{2}$  D. $\frac{\sqrt{2}}{4}$  E. NOTA

17. A triangular pasture is in the shape of a $30^\circ$-$60^\circ$-$90^\circ$ triangle (with a 6 m side opposite the $30^\circ$ angle) and is surrounded by fence along its two shorter sides, while the third side of the pasture (the “hypotenuse”) is unfenced. A goat is tethered by a 7 m long rope to a post at the corner of the pasture where the two fences meet. Over how many square meters can the goat graze inside the pasture if the goat starts off outside the fenced pasture area (and can’t jump over any fences)?

A. $\frac{16\pi}{3}$  B. $\pi$  C. $\frac{\pi}{6}$  D. $4\pi$  E. NOTA

18. If the vertices of an equilateral triangle have Cartesian coordinates $(0, 0), (a, 3\sqrt{3})$, and $(b, 0)$, find $ab$.

A. 6  B. 18  C. 54  D. 28  E. NOTA

19. Find the area of the triangle created by joining 3 non-adjacent vertices of a cube of side length 7.

A. $\frac{343\sqrt{3}}{6}$  B. $7\sqrt{6}$  C. $\frac{49\sqrt{3}}{2}$  D. $\frac{343\sqrt{6}}{9}$  E. NOTA
20. In the figure shown below, if the acute angle \( \theta \) is held fixed, any value for angle \( x \) between \( 0^\circ \) and \( (180 - \theta)^\circ \) (exclusive) will yield a triangle with one of its sides lying along \( \overline{AB} \).

If the angle \( x \) is chosen to have a random value between \( 0^\circ \) and \( (180 - \theta)^\circ \) (exclusive) for a fixed value of \( \theta \), what is the probability that the triangle formed will be acute?

A. \( \frac{90 + \theta}{180} \)  
B. \( \frac{\theta}{90 - \theta} \)  
C. \( \frac{\theta}{180 - \theta} \)  
D. \( \frac{90 + \theta}{180 - \theta} \)  
E. NOTA

21. In triangle ABC, the median from vertex A intersects its opposite side at D, the median from B intersects its opposite side at E, and the median from C does likewise at F. All three medians intersect at G. What is \( m\angle AGF + m\angle CGE + m\angle BGD \)?

A. 120°  
B. 180°  
C. 150°  
D. Can’t be determined  
E. NOTA

22. Consider rectangle ABCD with interior point E. Given the formula: \( \cos(180^\circ - \theta) = -\cos(\theta) \) and that \( m\angle AEB + m\angle DEC = 180^\circ \), AE = 1, BE = 7, and CE = 8, find the area of the rectangle.

A. 42  
B. 36  
C. 64  
D. 32  
E. NOTA

23. Line segments originating at point A (outside of circle C) are tangent to circle C at points B and D, determining an arc BD measuring 120°. Find the area of quadrilateral ABCD if the circle’s radius is 4.

A. \( 16\sqrt{3} \)  
B. \( \frac{16\sqrt{3}}{3} \)  
C. \( 12\sqrt{2} \)  
D. 24  
E. NOTA

24. A triangle has side lengths that form both an arithmetic and geometric sequence. If the triangle’s inradius is 2, find the area of its circumcircle.

A. \( 2\pi \)  
B. \( 4\pi \)  
C. \( 20\pi \)  
D. \( 16\pi \)  
E. NOTA
25. The lines parallel to side EF of triangle DEF divide DE into 6 equal line segments and likewise for DF. What fraction of triangle DEF’s area is shaded if EF is 40 and the altitude to EF is 30?

A. \( \frac{5}{11} \)  
B. \( \frac{7}{12} \)  
C. \( \frac{1}{2} \)  
D. \( \frac{2}{3} \)  
E. NOTA

26. A regular polygon is created by taking a pen to paper and tracing a straight line north for 6 units; then, turning in a direction 30° clockwise and tracing a line in that direction for 6 units; then turning 30° clockwise again and tracing in that direction for 6 units, and so on. When the polygon is completed, what will be its perimeter?

A. 72  
B. 54  
C. 27  
D. 96  
E. NOTA

27. For the polygon mentioned in Problem #26, what will be the sum of its interior angles?

A. 720°  
B. 1040°  
C. 1080°  
D. 2160°  
E. NOTA

28. What is the volume of a regular tetrahedron if the area of a face is \( 25 \sqrt{3} \)?

A. \( 100 \sqrt{3} \)  
B. \( 50 \sqrt{3} \)  
C. \( \frac{250 \sqrt{2}}{3} \)  
D. \( \frac{125 \sqrt{2}}{12} \)  
E. NOTA

29. If AD is the median to the hypotenuse of triangle ABC with sides 3, 4, 5 and AE is an altitude, find \( \sin \angle DAE \).

A. \( \frac{7}{25} \)  
B. \( \frac{24}{25} \)  
C. \( \frac{7}{24} \)  
D. \( \frac{\sqrt{3}}{3} \)  
E. NOTA

30. A regular hexagon PQRSTU has an inscribed circle of area \( 25\pi \). Find the sum of the lengths of the diagonals of PQRSTU.

A. \( 45\sqrt{2} \)  
B. \( 90 + 6\sqrt{3} \)  
C. \( 36\sqrt{6} \)  
D. \( 60 + 20\sqrt{3} \)  
E. NOTA