Alpha Bowl
Nationals 2008
Question #1

Consider the functions $f(x)=\dfrac{x^2-3x-4}{2x^2-x-3}$ and $g(x)=\begin{cases} x, & \text{if } x \text{ is a number} \\ 5, & \text{if } x \text{ is } +\infty \\ -5, & \text{if } x \text{ is} -\infty \end{cases}$.

$\lim\limits_{x\to -1^+} f(x)=A$ $\lim\limits_{x\to 4^+} f(x)=B$ $\lim\limits_{x\to 1.5^+} f(x)=C$ $\lim\limits_{x\to \infty} f(x)=D$

Evaluate: $g(\text{A})+2g(\text{B})+3g(\text{C})+4g(\text{D})$.

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Question #2

A = $\log_8(0.25)$

B = the greatest solution to the equation $\dfrac{x}{x-1}=\dfrac{x+4}{3}$

C = $\csc^2\left(\dfrac{5\pi}{3}\right)$

Evaluate: $\dfrac{AB}{C}$

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Question #3

Consider the function $f(\theta) = 4 - 3\sin\left(2\theta + \frac{\pi}{8}\right)$.

Let **A** be the amplitude of f.

Let **B** be the period of f.

Let **C** = $f\left(\frac{3\pi}{16}\right)$.

Evaluate ABC.

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Question #4

A = $\cos\theta$**,** if θ is the angle between the vectors <12,5> and $<8,6>$

B = $\tan\theta$**,** if θ is the smallest angle in a right triangle with one leg measuring 9 and hypotenuse measuring 41.

C = $\cot\theta$**,** if $\cos\theta = \sin(2\theta)$ and $0 < \theta < \frac{\pi}{2}$.

D = $\csc\theta$**,** if θ is the acute angle between the *x*-axis and the line with equation $2x - y = 3$.

Evaluate: $\frac{AC^2}{BD^2}$.

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Question #5

There is a box with socks and rocks.
There are 4 black socks and 3 white socks; there are 2 black rocks and 1 white rock.

I randomly draw **two** items from the box, and I **keep any black** item that I draw; I replace **any white item** that I draw.

A = the probability that I draw two black socks.

B = the probability that I draw differently colored items.

C = the probability that I draw a sock first, and then a rock.

D = the probability that I draw two white socks.

Evaluate $A+B+C+D$.

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Question #6

Consider the vectors $A = <2, -4>$ and $B = <-1, 5>$.

$2A - 4B = <m, n>$.

A is orthogonal to the vector $<5,\ p>$.

B is parallel to the vector $<5,\ t>$.

Find: $\frac{n}{m} - \frac{t}{p}$.

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Question #7

Given that $\log 2 \approx 0.3$ and $\ln(10) \approx 2 + \log 2$,

approximate $\log\left(\frac{1}{4}\right) + \ln(50)$ to the nearest hundredth.

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Question #8

A = the number of zeroes at the end of $6240!$

B = the number of distinct positive integers that are factors of 6240

C = the greatest perfect square that is less than 6240

D = 25% of 10% more than 6240

Evaluate: $A + B + C + D$

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Question #9

Consider the triangle JKL. Use the approximations given below to evaluate A, B, and C.

$\sin 50° \approx \frac{7}{9}$	$\sin 100° \approx \frac{49}{50}$
$\cos 50° \approx \frac{16}{25}$	$\cos 100° \approx -\frac{17}{100}$

A = JK**,** if $m\angle J = 50°$, $m\angle K = 30°$, and $KL = 14$.

B = JK**,** if $m\angle J = 50°$, $m\angle L = 30°$, and $KL = 14$.

C = JK**,** if $m\angle L = 50°$, $JL = 10$, and $KL = 14$.

Evaluate: $25A + B + 5C^2$

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Question #10

Consider the matrices $P = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$.

If $(PQ) + P^{-1} + Q^{\mathrm{T}} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then evaluate $a + d$.

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Question #11

If normal notation of a complex number is $a+bi$ where a and b are real, then set **R** is the set of all complex numbers where $a>b$ in normal notation.

Set **S** is the set of all solutions to the equation $x^{12}=1$.

Set **T** is the set $\left\{\operatorname{cis}\frac{\pi}{4},\operatorname{cis}\frac{5\pi}{6},\operatorname{cis}\frac{4\pi}{3},\operatorname{cis}\frac{3\pi}{2}\right\}$.

How many elements are in the set $(\mathbf{R}\cap(\mathbf{S}\cap\mathbf{T}))$?

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Question #12

Consider the functions $f(x)=3x-1$ and $g(x)=2x^2-2x-4$.

Let **A** = the distance between the points of intersection of the graphs of $y=f(x)$ and $y=g(x)$.

Let **B** = the value of k such that the graph of $y=g(x)-f(x)+k$ has exactly one x-intercept.

Let **C** = the remainder of the division $g(x)\div f(x)$.

Evaluate: $\dfrac{9BC\sqrt{10}}{A}$.