

NOTA means “None of the above”

1.  $\sin^{-1}\left(\cos\frac{19\pi}{6}\right)=?$

- a)  $\frac{-\pi}{3}$       b)  $\frac{-\pi}{6}$       c)  $\frac{4\pi}{3}$       d)  $\frac{5\pi}{3}$       e) NOTA

2. Find the cosine of the acute angle formed by vectors  $\langle 3, -2, 1 \rangle$  and  $\langle -1, 2, 5 \rangle$

- a)  $\frac{-\sqrt{420}}{210}$       b)  $\frac{-\sqrt{406}}{203}$       c)  $\frac{-\sqrt{105}}{105}$       d)  $\frac{2\sqrt{273}}{105}$       e) NOTA

3. Find the equation of the tangent line that intersects the circle  $x^2 + y^2 - 12x + 8y + 7 = 0$  at the point (3,2). Answer must be in  $Ax + By = C$  form, with  $A > 0$  and A, B, and C are relatively prime.

- a)  $x - 2y = -1$       b)  $3x - 2y = 5$       c)  $2x - y = 4$       d)  $2x + y = 8$       e) NOTA

4. The cable of a suspension bridge has supporting towers that are 24 yards high and 80 yards apart. The cable lies in a parabolic shape. If the lowest point of the cable is 4 yards above the floor of the bridge, find the height in yards of a supporting rod 30 yards from the center of the span.

- a)  $\frac{51}{4}$       b)  $\frac{25}{4}$       c)  $\frac{131}{8}$       d)  $\frac{61}{4}$       e) NOTA

5. Find the sum of the smallest and largest real solutions for:  $y^5 + 3y^4 - 3y^3 - 9y^2 - 4y - 12 = 0$ .

- a) 1      b) -5      c) -3      d) -1      e) NOTA

6. Nine Mu Alpha Theta competitors are getting ready for a ciphering competition. If 3 of the competitors are clearly superior competitors to the rest and are sure to be in the top three, in how many ways can the competition end?

- a) 720      b) 726      c) 4320      d) 12,960      e) NOTA

7. If  $\sin x + \cos x = k$ , evaluate  $\cos^2(2x)$  in terms of  $k$ .

- a)  $-2k^2$       b)  $-2k^2 - 2k$       c)  $-k^4 + 2k^2$       d)  $k^4 - 2k^2 + 1$       e) NOTA

8. If  $\tan \theta = \frac{3}{4}$ ,  $0 \leq \theta < \frac{\pi}{2}$  and  $\sin \beta = \frac{-5}{13}$ ,  $\pi \leq \beta < \frac{3\pi}{2}$ , find  $\cos(\theta + \beta)$ .

- a)  $\frac{-33}{165}$       b)  $\frac{-27}{65}$       c)  $\frac{33}{165}$       d)  $\frac{63}{65}$       e) NOTA

9. A vertical pole casts a shadow 12 units wide at one time and 4 units at a later time, when the angle of the line of sight to the sun and the horizontal is doubled. Find the height of the pole.

- a) 20      b) 15      c) 6      d)  $4\sqrt{3}$       e) NOTA

10. If  $X$  varies inversely as the square of  $Y$ . What happens to  $X$  if  $Y$  is cut in half?

- a) Divided by 4      b) Quadruples      c) Halved      d) Doubles      e) NOTA

11. The graph of  $g(x)$  has domain  $-4 \leq x \leq 4$  and range  $-3 \leq y \leq 2$ . What is the range of  $|g(x-2)| - 4$ ?

- a)  $[-7, -2]$       b)  $[-4, -1]$       c)  $[-4, 1]$       d)  $[-4, -2]$       e) NOTA

12. Solve for k:  $(1-i)^k = 4096$ .  $i = \sqrt{-1}$ .

- a) 12      b) 18      c) 24      d) 48      e) NOTA

13. Find the area of the convex quadrilateral with the following vertices: F(1,2), A(2,7), S(4,5), and T(-6,1).

- a) 23      b) 46      c) 3      d) 24      e) NOTA

14. Solve for x:  $2\left(e^{\ln(x+3)}\right)\left(\ln e^{(2x-3)}\right) = \frac{1}{2}e^{\ln(-6x+20)}$

- a) -4      b)  $\frac{7}{4}$       c)  $\frac{9}{4}$       d)  $\frac{-9+8\sqrt{7}}{8}$       e) NOTA

15. Find the slope of the line that passes through the intersection points of the curves  $y = 4\cos x$  and  $y = 2\csc x$ , where  $0 \leq x \leq 2\pi$

- a) 0      b)  $\frac{-4\sqrt{2}}{\pi}$       c)  $\frac{\pi}{4}$       d)  $\frac{-\pi}{4}$       e) NOTA

16. If  $\csc^2 \frac{\theta}{7} + \csc^2 \frac{2\theta}{7} + \csc^2 \frac{3\theta}{7} + \csc^2 \frac{4\theta}{7} = \frac{5}{7}$ , then what does

$$\cot^2 \frac{\theta}{7} + \cot^2 \frac{2\theta}{7} + \cot^2 \frac{3\theta}{7} + \cot^2 \frac{4\theta}{7} = ?$$

- a)  $\frac{-2}{7}$       b)  $\frac{-23}{7}$       c)  $\frac{23}{7}$       d)  $\frac{33}{7}$       e) NOTA

17. If  $\tan 2\theta = \frac{3}{4}$  for  $\left(\frac{\pi}{2} < \theta < \pi\right)$ , then the value of  $\cos \theta$  is?

- a)  $\frac{-3\sqrt{10}}{10}$       b)  $\frac{3\sqrt{10}}{10}$       c)  $\frac{-\sqrt{10}}{10}$       d)  $\frac{\sqrt{10}}{10}$       e) NOTA

18. If the coefficient of the 4<sup>th</sup> and 10<sup>th</sup> terms in the expansion of  $(w - f)^n$  are equal, find the 8<sup>th</sup> term.

- a)  $-3003w^6f^7$       b)  $-330w^5f^7$       c)  $-495w^5f^7$       d)  $-792w^5f^7$       e) NOTA

19. What is the sum of the real solutions to the equation:  $y^{\log y} = \frac{y^5}{1000000}$ ?

- a) 100      b) 1100      c) 100010      d) 1000000.1      e) NOTA

20. The latus rectum of the parabola with equation  $(x - 2)^2 = 12(y - 4)$  coincides with the diameter of a circle. If the equation of the circle is written in the form  $x^2 + y^2 + dx + ey + f = 0$ , what does  $d + e + f = ?$

- a) -28      b) -1      c) 26      d) 36      e) NOTA

21. Given a right triangle with legs equal to  $\log x$  and  $2\sqrt{\log x}$  and hypotenuse  $2\sqrt{3}$  with  $x > 1$ . What does  $\sin \theta$  equal if  $\theta$  is the angle opposite the side whose length is  $\log x$ ?

- a)  $-\sqrt{3}$       b)  $\sqrt{3}$       c)  $\frac{\sqrt{3}}{3}$       d)  $\frac{\sqrt{6}}{3}$       e) NOTA

22. Given  $\sin(4\theta) = \frac{8}{17}$  find  $(\cos \theta - \sin \theta)^2$ . (assume  $0 < \theta < 22.5$ ).

- a)  $\frac{-15}{17}$       b)  $\frac{15}{17}$       c)  $\frac{\sqrt{17}}{17}$       d)  $\frac{17 - \sqrt{17}}{17}$       e) NOTA

23. If  $\cot \phi = x$  and  $\cot \theta = y$ , find  $\cot(\phi - \theta)$  in terms of  $x$  and  $y$ , for  $xy \neq 0$ , and  $|x| \neq |y|$ .

- a)  $\frac{xy-1}{x+y}$       b)  $\frac{xy+1}{y-x}$       c)  $\frac{x-y}{xy+1}$       d)  $\frac{y-x}{xy+1}$       e) NOTA

24. Find the radius of the circle that passes through the points  $(1, -2)$ ,  $(5, 4)$ , and  $(10, 5)$ .

- a)  $\sqrt{82}$       b)  $\sqrt{65}$       c)  $\sqrt{29}$       d)  $2\sqrt{13}$       e) NOTA

25. If  $(x+3)$  is a factor of  $x^3 - x^2 - kx + 21$ . Which of the following is the product of the remaining roots?

- a) 7      b) -7      c) 5      d) -5      e) NOTA

26. Find  $n$  so that the function defined by  $f(x) = \frac{x+5}{x+n}$  will be its own inverse.

- a) -1      b) 1      c) 4      d) 5      e) NOTA

27.  $f$  is an even function,  $g$  and  $h$  are odd functions, all three are polynomials. Given:

$$f(1) = 0 \quad f(4) = 2 \quad f(3) = 6$$

$$g(1) = -1 \quad g(-2) = 4 \quad g(5) = 3$$

$$h(1) = 2 \quad h(3) = 5 \quad h(6) = 3$$

Find  $f(g(h(1))) + g^{-1}(h(f(-3))) + h(f(g(-1)))$

- a) -1      b) 1      c) -7      d) 7      e) NOTA

28. How many odd numbers between 450 and 700 can be formed using only the digits 3, 4, 5, 6, 7, and 8?

- a) 32      b) 37      c) 45      d) 48      e) NOTA

29. What is the product of the solutions:  $\log_{243} x - \log_x 9 = \frac{3}{5}$ .

- a)  $\frac{1}{9}$       b) 3      c) 27      d) 729      e) NOTA

30. The following table defines the operation  $\&$  on a set of four elements: 0, 1, 2, and 3. For example,  $1\&3 = 2$ , because 2 is in the box to the right of 1 and below 3 in the table. What is the identity element for  $\&$ ?

| $\&$ | 0 | 1 | 2 | 3 |
|------|---|---|---|---|
| 0    | 1 | 0 | 0 | 3 |
| 1    | 1 | 0 | 1 | 2 |
| 2    | 1 | 0 | 2 | 1 |
| 3    | 1 | 0 | 3 | 0 |

- a) 0      b) 1      c) 2      d) 3      e) NOTA