NOTA means “None of the above”

1. \( \sin^{-1}\left( \cos \frac{19\pi}{6} \right) =? \)
   a) \(-\frac{\pi}{3}\)  b) \(-\frac{\pi}{6}\)  c) \(\frac{4\pi}{3}\)  d) \(\frac{5\pi}{3}\)  e) NOTA

2. Find the cosine of the acute angle formed by vectors \(\langle 3,-2,1 \rangle \text{ and } \langle -1,2,5 \rangle\)
   a) \(-\frac{\sqrt{420}}{210}\)  b) \(-\frac{\sqrt{406}}{203}\)  c) \(-\frac{\sqrt{105}}{105}\)  d) \(\frac{2\sqrt{273}}{105}\)  e) NOTA

3. Find the equation of the tangent line that intersects the circle \(x^2 + y^2 - 12x + 8y + 7 = 0\) at the point \((3,2)\). Answer must be in \(Ax + By = C\) form, with \(A>0\) and \(A, B,\) and \(C\) are relatively prime.
   a) \(x - 2y = -1\)  b) \(3x - 2y = 5\)  c) \(2x - y = 4\)  d) \(2x + y = 8\)  e) NOTA

4. The cable of a suspension bridge has supporting towers that are 24 yards high and 80 yards apart. The cable lies in a parabolic shape. If the lowest point of the cable is 4 yards above the floor of the bridge, find the height in yards of a supporting rod 30 yards from the center of the span.
   a) \(\frac{51}{4}\)  b) \(\frac{25}{4}\)  c) \(\frac{131}{8}\)  d) \(\frac{61}{4}\)  e) NOTA

5. Find the sum of the smallest and largest real solutions for: \(y^5 + 3y^4 - 3y^3 - 9y^2 - 4y - 12 = 0\).
   a) 1  b) -5  c) -3  d) -1  e) NOTA
6. Nine Mu Alpha Theta competitors are getting ready for a ciphering competition. If 3 of the competitors are clearly superior competitors to the rest and are sure to be in the top three, in how many ways can the competition end?

a) 720  b) 726  c) 4320  d) 12,960  e) NOTA

7. If \( \sin x + \cos x = k \), evaluate \( \cos^2(2x) \) in terms of \( k \).

a) \(-2k^2\)  b) \(-2k^2 - 2k\)  c) \(-k^4 + 2k^2\)  d) \(k^4 - 2k^2 + 1\)  e) NOTA

8. If \( \tan \theta = \frac{3}{4}, 0 \leq \theta < \frac{\pi}{2} \) and \( \sin \beta = \frac{-5}{13}, \pi \leq \beta < \frac{3\pi}{2} \), find \( \cos(\theta + \beta) \).

a) \(-\frac{33}{165}\)  b) \(-\frac{27}{65}\)  c) \(\frac{33}{165}\)  d) \(\frac{63}{65}\)  e) NOTA

9. A vertical pole casts a shadow 12 units wide at one time and 4 units at a later time, when the angle of the line of sight to the sun and the horizontal is doubled. Find the height of the pole.

a) 20  b) 15  c) 6  d) \(4\sqrt{3}\)  e) NOTA

10. If \( X \) varies inversely as the square of \( Y \). What happens to \( X \) if \( Y \) is cut in half?

a) Divided by 4  b) Quadruples  c) Halved  d) Doubles  e) NOTA

11. The graph of \( g(x) \) has domain \(-4 \leq x \leq 4\) and range \(-3 \leq y \leq 2\). What is the range of \( |g(x - 2)| - 4 \)?

a) \([-7, -2]\)  b) \([-4, -1]\)  c) \([-4, 1]\)  d) \([-4, -2]\)  e) NOTA
12. Solve for \( k: (1-i)^k = 4096 \). \( i = \sqrt{-1} \).

a) 12 b) 18 c) 24 d) 48 e) NOTA

13. Find the area of the convex quadrilateral with the following vertices: F(1,2), A(2,7), S(4,5), and T(-6,1).

a) 23 b) 46 c) 3 d) 24 e) NOTA

14. Solve for \( x: 2 \left( e^{\ln(x+3)} \right) \left( \ln e^{(2x-3)} \right) = \frac{1}{2} e^{\ln(-x+20)} \)

a) -4 b) \( \frac{7}{4} \) c) \( \frac{9}{4} \) d) \( -\frac{9+8\sqrt{7}}{8} \) e) NOTA

15. Find the slope of the line that passes through the intersection points of the curves \( y = 4 \cos x \) and \( y = 2 \csc x \), where \( 0 \leq x \leq 2\pi \).

a) 0 b) \( -\frac{4\sqrt{2}}{\pi} \) c) \( \frac{\pi}{4} \) d) \( -\frac{\pi}{4} \) e) NOTA

16. If \( \csc^2 \frac{\theta}{7} + \csc^2 \frac{2\theta}{7} + \csc^2 \frac{3\theta}{7} + \csc^2 \frac{4\theta}{7} = \frac{5}{7} \), then what does \( \cot^2 \frac{\theta}{7} + \cot^2 \frac{2\theta}{7} + \cot^2 \frac{3\theta}{7} + \cot^2 \frac{4\theta}{7} = ? \)

a) \( -\frac{2}{7} \) b) \( -\frac{23}{7} \) c) \( \frac{23}{7} \) d) \( \frac{33}{7} \) e) NOTA
17. If \( \tan 2\theta = \frac{3}{4} \) for \( \frac{\pi}{2} < \theta < \pi \), then the value of \( \cos \theta \) is?

a) \( \frac{-3\sqrt{10}}{10} \)  

b) \( \frac{3\sqrt{10}}{10} \)  

c) \( \frac{-\sqrt{10}}{10} \)  

d) \( \frac{\sqrt{10}}{10} \)  

e) NOTA

18. If the coefficient of the 4th and 10th terms in the expansion of \((w - f)^n\) are equal, find the 8th term.

a) \(-300w^6f^7\)  

b) \(-330w^6f^7\)  

c) \(-495w^6f^7\)  

d) \(-792w^6f^7\)  

e) NOTA

19. What is the sum of the real solutions to the equation: \( y^{\log_{10} y} = \frac{y^5}{1000000} \)?

a) 100  

b) 1100  

c) 100010  

d) 1000000.1  

e) NOTA

20. The latus rectum of the parabola with equation \((x - 2)^2 = 12(y - 4)\) coincides with the diameter of a circle. If the equation of the circle is written in the form \( x^2 + y^2 + dx + ey + f = 0 \), what does \( d + e + f = ? \)

a) -28  

b) -1  

c) 26  

d) 36  

e) NOTA
21. Given a right triangle with legs equal to \(\log x\) and \(2\log x\) and hypotenuse \(2\sqrt{3}\) with \(x > 1\). What does \(\sin \theta\) equal if \(\theta\) is the angle opposite the side whose length is \(\log x\)?

a) \(-\sqrt{3}\)  
b) \(\sqrt{3}\)  
c) \(\frac{\sqrt{3}}{3}\)  
d) \(\frac{\sqrt{6}}{3}\)  
e) NOTA

22. Given \(\sin (4\theta) = \frac{8}{17}\) find \((\cos \theta - \sin \theta)^2\). (assume \(0 < \theta < 22.5\)).

a) \(\frac{-15}{17}\)  
b) \(\frac{15}{17}\)  
c) \(\frac{\sqrt{17}}{17}\)  
d) \(\frac{17 - \sqrt{17}}{17}\)  
e) NOTA

23. If \(\cot \phi = x\) and \(\cot \theta = y\), find \(\cot(\phi - \theta)\) in terms of \(x\) and \(y\), for \(xy \neq 0\), and \(|x| \neq |y|\).

a) \(\frac{xy - 1}{x + y}\)  
b) \(\frac{xy + 1}{y - x}\)  
c) \(\frac{x - y}{xy + 1}\)  
d) \(\frac{y - x}{xy + 1}\)  
e) NOTA

24. Find the radius of the circle that passes through the points \((1, -2)\), \((5, 4)\), and \((10, 5)\).

a) \(\sqrt{82}\)  
b) \(\sqrt{65}\)  
c) \(\sqrt{29}\)  
d) \(2\sqrt{3}\)  
e) NOTA

25. If \((x + 3)\) is a factor of \(x^3 - x^2 - kx + 21\). Which of the following is the product of the remaining roots?

a) 7  
b) -7  
c) 5  
d) -5  
e) NOTA

26. Find \(n\) so that the function defined by \(f(x) = \frac{x + 5}{x + n}\) will be its own inverse.

a) -1  
b) 1  
c) 4  
d) 5  
e) NOTA
27. \( f \) is an even function, \( g \) and \( h \) are odd functions, all three are polynomials. Given:
\[
\begin{align*}
  f(1) &= 0 & f(4) &= 2 & f(3) &= 6 \\
  g(1) &= -1 & g(-2) &= 4 & g(5) &= 3 \\
  h(1) &= 2 & h(3) &= 5 & h(6) &= 3
\end{align*}
\]
Find \( f(g(h(1))) + g^{-1}(h(f(-3))) + h(f(g(-1))) \)

a) -1  
 b) 1  
 c) -7  
 d) 7  
 e) NOTA

28. How many odd numbers between 450 and 700 can be formed using only the digits 3, 4, 5, 6, 7, and 8?

a) 32  
 b) 37  
 c) 45  
 d) 48  
 e) NOTA

29. What is the product of the solutions: \( \log_{243} x - \log_x 9 = \frac{3}{5} \).

a) \( \frac{1}{9} \)  
 b) 3  
 c) 27  
 d) 729  
 e) NOTA

30. The following table defines the operation \& on a set of four elements: 0, 1, 2, and 3. For example, \( 1 \& 3 = 2 \), because 2 is in the box to the right of 1 and below 3 in the table. What is the identity element for &?

\[
\begin{array}{c|cccc}
& 0 & 1 & 2 & 3 \\
\hline
0 & 1 & 0 & 0 & 3 \\
1 & 1 & 0 & 1 & 2 \\
2 & 1 & 0 & 2 & 1 \\
3 & 1 & 0 & 3 & 0 \\
\end{array}
\]

a) 0  
 b) 1  
 c) 2  
 d) 3  
 e) NOTA