National MAΘ 2008 **Mu Ciphering**

1. Let $f(x) = (x+2)(x+3)(x+4)$. Find the positive difference between $f'(1)$ and $f'(4)$.

2. If $\int_0^2 (2x^3 - kx^2 + 2k)\,dx = 12$, then evaluate $\int_{-k}^{k} \frac{dx}{x^2+9}$.

3. A square is inscribed in a circle. How fast is the area of the square increasing (in in^2/sec) when the area of the circle is increasing at $3\pi\ in^2/\text{sec}$?

4. Let c = the area of the region bounded by the graph of $y = 3x^2 + 3$ and the x-axis from $x = 1$ to $x = 3$. Evaluate $\int_0^c |x-1|\,dx$.

5. Let A = $\int_0^1 \frac{x\,dx}{x^2+1}$ and let B = $\int_{\pi/6}^{\pi/2} \cot x\,dx$. Express the positive difference between A and B as a single term.

6. Let the constant k equal the slope of the tangent to $y = \sin(xy) + \tan(2x)$ at the point $\left(\frac{\pi}{2}, 1\right)$. Find $f'(k^2)$ for $f(t) = \ln(t^k + k)$.

7. Given the curve defined parametrically by $x = t^2 - 1$ and $y = t^4 - 2t^3$ for $t > 0$, find $\frac{d^2y}{dx^2}$ when the graph intersects the x-axis.

8. Find the volume of the solid whose base is the region bounded by the ellipse $16x^2 + 9y^2 = 144$ and whose cross sections perpendicular to the x-axis are equilateral triangles.

9. Let line L be the line normal to the curve $y = \frac{1}{x^2}$ at the point (1,1). Find the area of the triangle formed by line L and the coordinate axes.

10. A cylindrical oil tank is 8 feet deep and 20 feet in diameter. At $t = 0$ minutes, the drain is opened so that the oil will flow out at a rate directly proportional to the square root of the volume of oil in the tank at any time. If the tank was full of oil initially, after how many minutes will it be empty if the oil initially flowed out at $20\pi\ ft^3/\text{min}$?

11. Let A = $\lim_{x\to 0} \frac{3\sin x}{e^x}$ and let B = $\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{xe^{2x}}\right)$ and let C = $\lim_{x\to o^+} x^2 \ln x$. Find the sum A + B + C.

12. Let R be the first quadrant region bounded by the graph of $y = x$ and $y = x^3$. Find the positive difference between the volumes generated by revolving R about the x-axis and by revolving R about the y-axis.