National MAΘ 2008 **Mu Ciphering Solutions**

1. $f(x) = (x+2)(x+3)(x+4)$. $f'(x) = (x+2)(x+3) + (x+2)(x+4) + (x+3)(x+4)$
$f'(1) = 3\bullet 4 + 3\bullet 5 + 4\bullet 5 = 12+15+20 = 47$
$f'(4) = 6\bullet 7 + 6\bullet 8 + 7\bullet 8 = 42+48+56 = 146$
$146 - 47 = 99$

2. $\int_0^2 (2x^3 - kx^2 + 2k)\,dx = 12 \Rightarrow \frac{1}{2}x^4 - \frac{k}{3}x^3 + 2kx\Big]_0^2 = 12 \Rightarrow 8 - \frac{8}{3}k + 4k = 12 \Rightarrow k = 3.$
$\int_{-3}^{3} \frac{dx}{x^2+9} = \frac{1}{3}\arctan\frac{x}{3}\Big]_{-3}^{3} = \frac{1}{3}\arctan 1 - \frac{1}{3}\arctan(-1) = \frac{1}{3}\bullet\frac{\pi}{4} - \frac{1}{3}\bullet\frac{-\pi}{4} = \frac{\pi}{6}$.

3. $A_{circle} = \pi r^2$; $\frac{dA_{circle}}{dt} = 2\pi r\frac{dr}{dt} = 3\pi \Rightarrow r\frac{dr}{dt} = \frac{3}{2}$. $A_{sq} = 2r^2$; $\frac{dA_{sq}}{dt} = 4r\frac{dr}{dt} = 4\bullet\frac{3}{2} = 6$

4. $A = \int_1^3 (3x^2+3)\,dx = (x^3+3x)\Big]_1^3 = 27+9-4 = 32$. $\int_0^{32} |x-1|\,dx = \frac{1}{2} + \frac{961}{2} = 481.$

5. A = $\int_0^1 \frac{x\,dx}{x^2+1} = \frac{1}{2}\ln(x^2+1)\Big]_0^1 = \frac{1}{2}\ln 2$; B = $\int_{\pi/6}^{\pi/2} \cot x\,dx = \ln|\sin x|\Big]_{\pi/6}^{\pi/2} = 0 - \ln\frac{1}{2} = \ln 2$
$\ln 2 - \frac{1}{2}\ln 2 = \frac{1}{2}\ln 2$ or $\ln\sqrt{2}$

6. $y = \sin(xy) + \tan(2x)$; $y' = (xy' + y)\cos(xy) + 2\sec^2(2x)$. At the point $\left(\frac{\pi}{2}, 1\right)$, $y' = 2\sec^2\pi = 2$.
$f(t) = \ln(t^2+2)$ and $f'(t) = \frac{2t}{t^2+2}$ and so $f'(4) = \frac{4}{9}$

7. $\frac{dy}{dx} = \frac{4t^3 - 6t^2}{2t} = 2t^2 - 3t$. $\frac{d^2y}{dx^2} = \frac{4t-3}{2t}$. The graph intersects the x-axis when t = 2 and so $\frac{d^2y}{dx^2} = \frac{5}{4}$

8. $16x^2 + 9y^2 = 144 \Rightarrow y = \pm\sqrt{16 - \frac{16}{9}x^2}$ and $s = 2\sqrt{16 - \frac{16}{9}x^2}$ and so $A = \frac{s^2\sqrt{3}}{4}$ and
$V = 2\int_0^3 \sqrt{3}(16 - \frac{16}{9}x^2)\,dx = 2\sqrt{3}(16x - \frac{16}{27}x^3)\Big]_0^3 = 2\sqrt{3}(48-16) = 64\sqrt{3}$

9. $y = \frac{1}{x^2}$; $\frac{dy}{dx} = \frac{-2}{x^3}$ at the point (1,1) gives m = -2. Equation of L is $y - 1 = \frac{1}{2}(x-1)$ or $y = \frac{1}{2}x + \frac{1}{2}$.
Area of triangle is $\frac{1}{2}\bullet 1\bullet\frac{1}{2} = \frac{1}{4}$

10. When $t = 0, V = 800\pi$. $\frac{dV}{dt} = k\sqrt{V} \Rightarrow \int V^{-1/2}\,dV = k\int dt \Rightarrow 2\sqrt{V} = kt + C$ and so $C = 40\sqrt{2\pi}$.
Also, since $\frac{dV}{dt} = k\sqrt{V}$, then $-20\pi = k\sqrt{800\pi} \Rightarrow k = \frac{-\sqrt{\pi}}{\sqrt{2}}$. So now $2\sqrt{V} = \frac{-\sqrt{\pi}}{\sqrt{2}}t + 40\sqrt{2\pi}$ and
let $V = 0$ and solve for t. $\frac{\sqrt{\pi}}{\sqrt{2}}t = 40\sqrt{2\pi} \Rightarrow t = 40\sqrt{2\pi}\bullet\frac{\sqrt{2}}{\sqrt{\pi}} = 80$

11. A = $\lim_{x\to 0} \frac{3\sin x}{e^x} = \frac{3\bullet 0}{1} = 0$. B = $\lim_{x\to 0}(\frac{1}{x} - \frac{1}{xe^{2x}}) = \lim_{x\to 0}(\frac{e^{2x}-1}{xe^{2x}}) = \lim_{x\to 0}(\frac{2e^{2x}}{2xe^{2x}+e^{2x}}) = \frac{2}{1} = 2$.

C = $\lim_{x\to o^+} x^2 \ln x = \lim_{x\to o^+} \frac{\ln x}{x^{-2}} = \lim_{x\to o^+} \frac{1/x}{-2x^{-3}} = \lim_{x\to o^+} \frac{1}{-2x^{-2}} = \lim_{x\to o^+} \frac{-1}{2}x^2 = 0$. A + B + C = 0 + 2 + 0 = 2.

12. Revolving R about the x-axis: $V = \pi\int_0^1 (x^2 - x^6)\, dx = \pi(\frac{1}{3} - \frac{1}{7}) = \frac{4\pi}{21}$.

Revolving R about the y-axis: $V = 2\pi\int_0^1 x(x - x^3)\, dx = 2\pi\int_0^1 (x^2 - x^4)\, dx = 2\pi(\frac{1}{3} - \frac{1}{5}) = \frac{4\pi}{15}$.

$\frac{4\pi}{15} - \frac{4\pi}{21} = \frac{28\pi - 21\pi}{105} = \frac{8\pi}{105}$