NATIONALS 2008
MU GEMINI

For all questions, choice (E) is NOTA, meaning "None of these answers."

1. Evaluate: $\lim_{y\to 0}\dfrac{\cos(1.5\pi + y)}{y}$.

A) -1 B) 0 C) 1 D) $\dfrac{\sqrt{2}}{2}$ E) NOTA

2. If $\dfrac{dy}{dt} = -3y$ and $y = 1$ when $t = 0$, then find t when $y = \dfrac{1}{3}$.

A) -3 B) $-\dfrac{\ln 3}{3}$ C) $-\dfrac{1}{3}$ D) $\dfrac{\sqrt{3}}{3}$ E) NOTA

3. If $h(x) = \dfrac{1}{\sqrt[5]{x^4}}$ and $h'(x) = ax^n$, then find the tenths digit of $|a+n|$.

A) 0 B) 2 C) 5 D) 6 E) NOTA

4. Consider the hyperbola with equation $\dfrac{(y-3)^2}{20} - \dfrac{(x+1)^2}{5} = 1$. Let a = the y-intercept of the asymptote with positive slope. Let b = the sum of the y-coordinates of the foci. Evaluate $|a-b|$.

A) 1 B) 2 C) 3 D) 4 E) NOTA

5. Let $W = f''\left(\dfrac{\pi}{4}\right)$ when $f(x) = \dfrac{1}{\cos x}$. Evaluate W^2.

A) 6 B) 8 C) 12 D) 18 E) NOTA

6. Evaluate: $\int_0^1 x^3 e^{x^4}\,dx$.

A) $\dfrac{e-1}{4}$ B) $\dfrac{e}{4}$ C) $4(e-1)$ D) $4e^4 - 1$ E) NOTA

NATIONALS 2008
MU GEMINI

7. A dog is tied to the corner of a regular hexagonal shed that stands on $54\sqrt{3}$ square feet. If the rope with which the dog is tied is 12 feet long, then the area outside the shed that is accessible to the dog is $k\pi$ square feet. Find the units digit of *k*.

A) 0 B) 4 C) 6 D) 8 E) NOTA

8. The position of a particle on the number line for all times $t \geq 0$ is given by the equation $s(t) = \frac{1}{3}x^3 - 4x^2 + 15x - 180$. When is the particle's velocity increasing?

A) $0 < t < 3$ and $t > 5$ B) $3 < t < 5$ C) $0 < t < 4$ D) $t > 4$ E) NOTA

9. Consider the arithmetic shown, where each digit is represented by exactly one distinct letter. Find the units digit of $(F + I + T)$.

$$\begin{array}{cccccccc} & & F & I & F & T & Y \\ + & S & T & A & T & E & S \\ \hline A & M & E & R & I & C & A \end{array}$$

A) 6 B) 7 C) 8 D) 9 E) NOTA

10. Find the slope of the line tangent to the circle $x^2 + y^2 - 4x - 6y = 12$ at its positive *x*-intercept.

A) -2 B) $\frac{4}{3}$ C) 2 D) 10 E) NOTA

11. A box contains 3 green marbles, 2 blue marbles, and 4 red marbles. If you draw two marbles at random (without putting the first back), then $\frac{x}{72}$ is the probability that they are different colors. Find the sum of the digits of *x*.

A) 5 B) 6 C) 7 D) 8 E) NOTA

12. Let *S* be the sum of the <u>distinct</u> roots of the function $g(x) = x^4 - 2x^3 - 3x^2 + 4x + 4$. Evaluate $|S|$.

A) 1 B) 2 C) 3 D) 4 E) NOTA

NATIONALS 2008
MU GEMINI

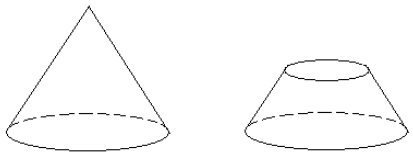
13. Find the units digit of $k(200)$, if $k(x)=\begin{cases}3, & \text{if } x\le 2\\ 2k(x-1), & \text{if } 2<x\le 5\\ k(x-2)+5, & \text{if } x>5\end{cases}$.

A) 2 B) 4 C) 7 D) 9 E) NOTA

14. Let $f(x)$ be a continuous function on $[a,b]$. If $f(b)<0<f(a)$, then which of the following **must** be greater than $\int_a^b f(x)\,dx$?

A) $\int_b^a f(x)\,dx$ B) $\int_a^b [f(x)]^2\,dx$ C) $\left|\int_a^b f(x)\,dx\right|$ D) $\int_a^b |f(x)|\,dx$ E) NOTA

15. A cone begins eroding from its apex. As it disintegrates continuously, it maintains the shape of a frustum (a truncated cone, as shown.) The radius of the original cone is 5 ft, and its height is 10 ft. When the frustum's height is 3 ft, its height is decreasing at a rate of $2\frac{\text{ft}}{\text{min}}$. At what rate is the frustum's volume decreasing at the same time?



A) $6\pi\frac{\text{ft}^3}{\text{min}}$ B) $\frac{16\pi}{3}\frac{\text{ft}^3}{\text{min}}$ C) $12\pi\frac{\text{ft}^3}{\text{min}}$ D) $\frac{49\pi}{2}\frac{\text{ft}^3}{\text{min}}$ E) NOTA

16. Evaluate: $\int_0^1 \frac{4x^3-6x}{x^4-3x^2-4}\,dx$

A) $\ln\left(\frac{3}{2}\right)$ B) Arcsin(1) C) $\sqrt{3}$ D) $\frac{1}{2}$ E) NOTA

17. Let f be a function, continuous and twice-differentiable over all real numbers, where $f'(k)=-(4-P)+1$ for some k on the interval $(-3,7)$, and $f'(-3)=f'(7)=2$. Which of the following conditions guarantees that f has a local maximum?

A) $P=3$ B) $P<3$ C) $3<P\le 5$ D) $P>5$ E) NOTA

NATIONALS 2008
MU GEMINI

18. Eleven lines are drawn in a plane in such a way that every pair of lines intersects, but no more than two lines intersect at any given point. The plane is as a result divided into *r* regions. Find the units digit of *r*.

A) 1 B) 3 C) 5 D) 7 E) NOTA

Use the table show below for questions #19 and #20.

x	1	2	3	4	5
$f(x)$	2	3	5	8	10
$f'(x)$	0.25	0.4	1.25	2.5	3.3

19. If $g(x)$ and $f(x)$ are inverse functions, both defined on the interval [1,5], then find $g'(3)$.

A) 0.8 B) 1.25 C) 2.5 D) 3.3 E) NOTA

20. If $h(x)=\dfrac{1}{f^2(x)}$, then evaluate $h'(1)$.

A) -16 B) -8 C) -4 D) -2 E) NOTA

21. If $\int_1^4 f(x)\,dx=6$ then evaluate $\int_1^4 f(5-x)\,dx$.

A) -6 B) -1 C) 6 D) 11 E) NOTA

22. Let $f(x)=\begin{cases}2x+1, & \text{if } x\le 2\\ 5, & \text{if } x>2\end{cases}$, and $g(x)=\int_0^x f'(t)\,dt$. Evaluate $g(3)$.

A) 0 B) 2 C) 4 D) 7 E) NOTA

23. $\sqrt{11-6\sqrt{2}}=x-y\sqrt{2}$, where *x* and *y* are rational. Evaluate $|x-y|$.

A) 1 B) 2 C) 3 D) 4 E) NOTA

24. Evaluate: $\lim_{x\to 0}\dfrac{1-\cos x}{x\sin x}$.

A) 0 B) $\dfrac{1}{2}$ C) 1 D) π E) NOTA

NATIONALS 2008
MU GEMINI

25. The curve in the *xy*-plane described by the parametric equations $x = \sin^2 t$ and $y = \sin t$ for $0 \le t \le \pi$ is a portion of which kind of graph?

A) a circle B) a line C) a parabola D) a sinusoid E) NOTA

26. Find the area of the region bounded by the graphs of $y = x^2$ and $y = x$.

A) $\frac{1}{6}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) NOTA

27. There are five contestants left on Super Funtime Money Quizshow – two men and three women. Each week, one contestant will be eliminated until only two are left. If the eliminations are completely random, then what is the probability that the two men will be the final two contestants?

A) $\frac{1}{12}$ B) $\frac{1}{10}$ C) $\frac{1}{8}$ D) $\frac{1}{5}$ E) NOTA

28. Find the value of *x* which satisfies the conclusion of the Mean Value Theorem for the function $y = x^2 - 3x - 1$ over the interval $[0, 4]$.

A) 0 B) 1 C) 2 D) 3 E) NOTA

29. For what non-negative value of *b* is the line $y = \frac{b - x}{3}$ normal to the graph of $f(x) = x^3$?

A) 0 B) 1 C) 4 D) 10 E) NOTA

30. A crate on a parachute is falling directly toward the ground at a constant rate of 30 km per hour. A man standing 4 km from the drop spot (the point on the ground where the crate will land) is watching it fall. At what rate (in radians per hour) is the angle of elevation from the bottom of the man's shoe to the crate decreasing when the crate is 3 km high?

A) $\frac{26}{25}$ B) $\frac{27}{10}$ C) $\frac{175}{64}$ D) $\frac{24}{5}$ E) NOTA