1. Let $f(x) = 2x^3 + 3x^2 - 12x + 1$. If m and M are the minimum and maximum values, respectively, of $f(x)$ on the interval $[-1, 2]$, then what is $M - m$?

   A. 9 B. 16 C. 20 D. 27 E. NOTA

2. Find the average value of $f(x) = \dfrac{\ln x}{x}$ on the interval $[1, 4]$.

   A. $\dfrac{1}{2}\ln 4$ B. $\dfrac{1}{2}(\ln 4)^2$ C. $\dfrac{1}{4}(\ln 4)^2$ D. $\dfrac{1}{6}(\ln 4)^2$ E. NOTA

3. Evaluate $\displaystyle\int_{\pi/2}^{2\pi} \frac{\sin\theta}{\sqrt{3+\cos\theta}}\, d\theta$.

   A. $4-\sqrt{3}$ B. $4+\sqrt{3}$ C. $4-2\sqrt{3}$ D. $4+2\sqrt{3}$ E. NOTA

4. Two real numbers *x* and *y* are chosen randomly from the intervals $[0, 2]$ and $[0, 5]$, respectively. Find the probability that $y - x^2 < 1$.

   A. $\dfrac{4}{15}$ B. $\dfrac{7}{15}$ C. $\dfrac{8}{15}$ D. $\dfrac{11}{15}$ E. NOTA

5. If $x + y = \ln x + xy + y^2$, then what is the value of $\dfrac{dy}{dx}$ at the point $(1, -1)$?

   A. -1 B. $-1/2$ C. $1/2$ D. 1 E. NOTA

6. Evaluate $\displaystyle\int_{\pi}^{2\pi} e^x \cos x\, dx$.

   A. $\dfrac{e^{2\pi} + e^{\pi}}{2}$ B. $-\dfrac{e^{2\pi} + e^{\pi}}{2}$ C. $\dfrac{e^{2\pi} - e^{\pi}}{2}$ D. $\dfrac{e^{\pi} - e^{2\pi}}{2}$ E. NOTA

7. The volume of a sphere is increasing at a rate of $12\ \text{m}^3/\text{sec}$. At what rate is the sphere's surface area increasing when the radius is $3\ \text{m}$?

   A. $8/9\ \text{m}^2/\text{sec}$ B. $8\ \text{m}^2/\text{sec}$ C. $12\ \text{m}^2/\text{sec}$ D. $18\ \text{m}^2/\text{sec}$ E. NOTA

8. If $f(2) = 3$, $f(3) = 5$, $f'(2) = 4$, $f'(3) = 2$ and $g(x) = f^{-1}(x)$, then what is $g'(3)$?

   A. $1/5$ B. $1/4$ C. $1/3$ D. $1/2$ E. NOTA

9. The region bounded by the curves $y = \sqrt{x}$, $y = 2$, and $x = 0$ is rotated about the x-axis. Find the volume of the resulting solid.

A. 2π B. $\frac{8\pi}{3}$ C. 6π D. 8π E. NOTA

10. How many of the following functions are differentiable everywhere?

$y = |x|$ $y = x|x|$ $y = |\sin x|$ $y = |x^3|$

A. 0 B. 1 C. 2 D. 3 E. NOTA

11. Evaluate: $\lim_{x\to\infty} 2x \sin\left(\frac{2}{x}\right)$

A. 0 B. 1 C. 4 D. ∞ E. NOTA

12. Use differentials to best approximate the value of $\sqrt{51}$.

A. $\frac{29}{4}$ B. $\frac{51}{7}$ C. $\frac{115}{16}$ D. $\frac{141}{16}$ E. NOTA

13. Approximate the value of $\int_0^6 x^2 dx$ using a midpoint Riemann Sum with 3 equal subdivisions.

A. 55 B. 70 C. 72 D. 76 E. NOTA

14. Starting with $x_1 = 1$, use Newton's method to find the approximation x_3 to the root of $f(x) = x^3 - x + 1$.

A. -3 B. -1 C. 3 D. $7/2$ E. NOTA

15. If f is a function differentiable everywhere, and if $f(1) = 3$ and $f(4) = 9$, then what value of $f'(x)$ is guaranteed for some value of *x* between 1 and 4?

A. $\frac{2}{5}$ B. $\frac{1}{2}$ C. 2 D. $\frac{5}{2}$ E. NOTA

16. If $f(x) = \sum_{n=2}^{\infty} \frac{x^{3n+1}}{n!}$, then what is the value of $f(2)$?

A. $2e^8 - 18$ B. $2e^8 - 16$ C. $2e^8 - 1$ D. $2e^8$ E. NOTA

17. Let A be the area enclosed by the region of the curve $r^3 = \theta^6$ that lies in the sector $0 \le \theta \le \pi/2$. If $A = \dfrac{\pi^n}{m}$, where n, m are positive integers, then find m/n.

A. 8 B. 16 C. 32 D. 64 E. NOTA

18. Evaluate $\displaystyle\int_3^6 \frac{1}{\sqrt{x-3}}\,dx$.

A. $-\dfrac{\sqrt{3}}{2}$ B. $\dfrac{\sqrt{3}}{2}$ C. $-2\sqrt{3}$ D. $2\sqrt{3}$ E. NOTA

19. An ellipse centered at the origin has a horizontal major axis of length 8 and a vertical minor axis of length 4. What is the slope of the tangent line at the point $\left(2\sqrt{2}, \sqrt{2}\right)$?

A. $-\dfrac{1}{2}$ B. $\dfrac{1}{2}$ C. -1 D. 1 E. NOTA

20. Evaluate $\displaystyle\lim_{x\to 2} \frac{x^2 - x - 2}{x^2 - 3x + 2}$.

A. 0 B. 1 C. 3 D. ∞ E. NOTA

21. Evaluate $\displaystyle\lim_{x\to\infty} \left(\frac{x-e}{x}\right)^x$.

A. e^{-e} B. e^{e} C. e^{-1} D. e E. NOTA

22. Find the length of curve $f(x) = -\ln|\cos x|$ on the interval $\left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right]$.

A. 2 B. $\pi\sqrt{2} - \pi$ C. $\ln\left(3 - 2\sqrt{2}\right)$ D. $\ln\left(3 + 2\sqrt{2}\right)$ E. NOTA

23. Let $g(x) = \displaystyle\int_{-1}^{x} e^{f(t)}\,dt$. If f is a real-valued function, then how many solutions does $g(x) = 0$ have?

A. 0 B. 1
C. 2 D. cannot be determined E. NOTA

24. If $F(x)=\int_{1/2}^{x^2}\sin^2\left(t\cdot\frac{\pi}{3}\right)dt$, then what is the value of $F'(2)$?

A. $1/2$ B. $3/4$ C. $11/4$ D. 3 E. NOTA

25. A particle is moving along a curve on the coordinate plane. Its position can be given by the equations $x(t)=t^2+t+1$ and $y(t)=\ln t+2$. What is the particle's speed at $t=1$?

A. $1/3$ B. 3 C. $\sqrt{13}$ D. 9 E. NOTA

26. The curves $f(x)=1/x$ and $g(x)=-x^2$ have a common tangent line. If the line is tangent at the points $\left(a, f\left(a\right)\right)$ and $\left(b, g\left(b\right)\right)$, then find the value of $a+b$.

A. $3/2$ B. $5/2$ C. $7/2$ D. $9/2$ E. NOTA

27. If $f(x)=\left(x-3\right)^3\left(x-2\right)$, find $f'(5)$.

A. 13 B. 32 C. 44 D. 60 E. NOTA

28. Evaluate $\int_{-2}^{2}|x+2|\,dx$.

A. 4 B. 6 C. 8 D. 12 E. NOTA

29. Which pair of words results in a true statement when placed in the blanks?
If a function f is _____ everywhere, then f is also _____ everywhere.

A. differentiable … integrable B. integrable … continuous
C. continuous … differentiable D. integrable … differentiable E. NOTA

30. If $f(x)=x^5+x+1$, then what is the value of $\int_{1}^{3}f^{-1}(x)\,dx$?

A. $\frac{3}{5}$ B. $\frac{3}{4}$ C. $\frac{4}{3}$ D. $\frac{5}{3}$ E. NOTA