1. C	**6. A**	**11. C**	**16. A**	**21. A**	**26. B**
2. D	**7. B**	**12. E**	**17. D**	**22. D**	**27. C**
3. E	**8. B**	**13. B**	**18. D**	**23. B**	**28. C**
4. B	**9. D**	**14. C**	**19. A**	**24. D**	**29. A**
5. B	**10. C**	**15. C**	**20. C**	**25. E**	**30. C**

1. Taking the derivative, solving for 0, and testing intervals gives us that the maximum and minimum are $(-2,21)$ and $(1,-6)$, respectively. $x=-2$ is outside of our interval. So we test the endpoints. $f(-1)=14$ and $f(2)=5$. So $m=-6$. Since f is decreasing from $[-2,1]$, $M=14$. So $M-m=14+6=20$.

2. $\frac{1}{4-1}\int_1^4 \frac{\ln x}{x}dx=\frac{1}{3}\int_0^{\ln 4} udu=\frac{1}{3}\cdot\frac{1}{2}u^2\Big|_0^{\ln 4}=\frac{1}{6}(\ln 4)^2$

3. $\int_{\pi/2}^{2\pi}\frac{\sin\theta}{\sqrt{3+\cos\theta}}\,d\theta=-2\sqrt{3+\cos\theta}\Big|_{\pi/2}^{2\pi}=-4+2\sqrt{3}$

4. We can think of this as the probability of picking a point within the region bounded by the x – axis, y – axis, and $x=2$ from the rectangle bounded by the x – axis, y – axis, $x=2$ and $y=5$. The area of the rectangle is 10. The area under the region is $\int_0^2 1+x^2dx=x+\frac{x^3}{3}\Big|_0^2=2+\frac{8}{3}=\frac{14}{3}$. So the probability is $\frac{14}{3}\div 10=\frac{7}{15}$

5. Taking derivatives, we get: $1+y'=\frac{1}{x}+y+xy'+2y'$. Plugging in x and y and solving for y' yields $y'=1/2$.

6. $u=e^x,\ du=e^xdx,\ v=\sin x,\ dv=\cos xdx$. $\int e^x\cos xdx=\int udv=uv-\int vdu=e^x\sin x-\int e^x\sin xdx$
$u=e^x,\ du=e^xdx,\ v=-\cos x,\ dv=\sin xdx$
$\int e^x\sin xdx=\int udv=uv-\int vdu=-e^x\cos x+\int e^x\cos xdx$
So $\int e^x\cos xdx=e^x\sin x-\left(-e^x\cos x+\int e^x\cos xdx\right)$. So:
$\int e^x\cos xdx=\frac{1}{2}e^x(\sin x+\cos x)$ and $\int_\pi^{2\pi}e^x\cos x=\frac{1}{2}e^x(\sin x+\cos x)\Big|_\pi^{2\pi}=\frac{1}{2}\left(e^{2\pi}+e^\pi\right)$

7. $V=(4/3)\pi r^3\Rightarrow\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$. So $12=4\pi 3^2\frac{dr}{dt}\Rightarrow\frac{dr}{dt}=\frac{1}{3\pi}$.
$A=4\pi r^2\Rightarrow\frac{dA}{dt}=8\pi r\frac{dr}{dt}$. So $\frac{dA}{dt}=8\pi\cdot 3\frac{1}{3\pi}=8$.

8. $g'(3)=1/f'(g(3))=1/f'(2)=1/4$

9. $V=\pi\int_0^4\left(2^2-x\right)dx=4\pi x-\frac{\pi}{2}x^2\Big|_0^4=8\pi$

10. $y=|x|$ and $y=|\sin x|$ are not differentiable at $x=0$.

11. $\lim_{x\to\infty}2x\sin\left(\frac{2}{x}\right)=\lim_{x\to\infty}\frac{\sin(2/x)}{1/(2x)}$. Top and bottom go to 0, so by L'hopital's,
$\lim_{x\to\infty}\frac{\sin(2/x)}{1/(2x)}=\lim_{x\to\infty}\frac{-\frac{2}{x^2}\cos(2/x)}{-\frac{1}{2x^2}}=\lim_{x\to\infty}4\cos(2/x)=4$

12. Let $f(x)=\sqrt{x}$. Then $dy=\dfrac{dx}{2\sqrt{x}}$ Then let $x=49, dx=51-49=2$. $dy=\dfrac{2}{2\cdot 7}=\dfrac{1}{7}$.

$f(51)\approx f(49)+dy=7+1/7=50/7$.

13. $\int_0^6 x^2dx \approx \dfrac{6-0}{3}\left(\left(\dfrac{0+2}{2}\right)^2+\left(\dfrac{2+4}{2}\right)^2+\left(\dfrac{4+6}{2}\right)^2\right)=2\left(1+9+25\right)=70$

14. Newton's Method: $x_{n+1}=x_n-\dfrac{f(x_n)}{f'(x_n)}$. So $x_2=1-\dfrac{1}{2}=\dfrac{1}{2}$. $x_3=\dfrac{1}{2}-\dfrac{\frac{1}{8}-\frac{1}{2}+1}{\frac{3}{4}-1}=3$

15. From the Mean Value Theorem, there exists a c in $(1,4)$ such that $\dfrac{f(4)-f(1)}{4-1}=f'(c)$.

So $f'(c)=6/3=2$.

16. $f(x)=\sum_{n=2}^{\infty}\dfrac{x^{3n+1}}{n!}=\dfrac{x^7}{2!}+\dfrac{x^{10}}{3!}+\dfrac{x^{13}}{4!}+\cdots=x\left(\dfrac{\left(x^3\right)^2}{2!}+\dfrac{\left(x^3\right)^3}{3!}+\dfrac{\left(x^3\right)^4}{4!}+\cdots\right)=x\left(e^{x^3}-1-x^3\right)$

So $f(2)=2\left(e^8-9\right)=2e^8-18$

17. $r=\theta^2$. So $A=\dfrac{1}{2}\int_0^{\pi/2} r^2d\theta=\dfrac{1}{2}\int_0^{\pi/2}\theta^4d\theta=\dfrac{1}{10}\theta^5\Big|_0^{\pi/2}=\dfrac{\pi^5}{320}$. So $m/n=64$.

18. $\int_3^6 \dfrac{1}{\sqrt{x-3}}dx=\lim_{t\to 3}2\sqrt{x-3}\Big|_t^6=2\sqrt{3}$

19. The equation of the ellipse: $\dfrac{x^2}{16}+\dfrac{y^2}{4}=1$. Differentiating, we get: $\dfrac{x}{8}+\dfrac{yy'}{2}=0\Rightarrow y'=-\dfrac{x}{4y}$.

So at the specified point, $y'=-\dfrac{2\sqrt{2}}{4\sqrt{2}}=-\dfrac{1}{2}$.

20. $\lim_{x\to 2}\dfrac{x^2-x-2}{x^2-3x+2}=\lim_{x\to 2}\dfrac{(x-2)(x+1)}{(x-1)(x-2)}=\lim_{x\to 2}\dfrac{(x+1)}{(x-1)}=3$

21. $y=\left(\dfrac{x-e}{x}\right)^x\Rightarrow \ln y=x\ln\left(\dfrac{x-e}{x}\right)\Rightarrow \lim_{x\to\infty}\ln y=\lim_{x\to\infty}x\ln\left(\dfrac{x-e}{x}\right)=\lim_{x\to\infty}\dfrac{\ln\left(1-e/x\right)}{1/x}$.

Top and bottom go to 0, so using L'hopital's:

$\lim_{x\to\infty}\ln y=\lim_{x\to\infty}\dfrac{\left(e/x^2\right)/\left(1-e/x\right)}{-1/x^2}=\lim_{x\to\infty}\dfrac{e}{e/x-1}=-e$. So $\lim_{x\to\infty}\ln y=-e\Rightarrow\lim_{x\to\infty}y=e^{-e}$.

22.

$$\begin{aligned}\int_a^b\sqrt{1+\left(dy/dx\right)^2}dx=\int_{-\pi/4}^{\pi/4}\sqrt{1+\tan^2 x}dx &= \int_{-\pi/4}^{\pi/4}\sqrt{\sec^2 x}dx\\ &= \int_{-\pi/4}^{\pi/4}\sec x dx\\ &= \ln(\sec x+\tan x)\Big|_{-\pi/4}^{\pi/4}\\ &= \ln\left(3+2\sqrt{2}\right)\end{aligned}$$

23. Clearly, $g(x)=0$ when $x=-1$. $g'(x)=e^{f(x)}>0$, so $g(x)$ is always increasing. So $g(x)$ is 0 only at $x=-1$.

24. $F'(x)=2x\sin^2\left(x\cdot\frac{\pi}{3}\right)$. So $F'(2)=4\sin^2\left(2\cdot\frac{\pi}{3}\right)=3$

25. $v(t)=\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}=\sqrt{(2t+1)^2+\left(\frac{1}{t}\right)^2}$. So $v(1)=\sqrt{10}$

26. $f'(a)=g'(b)\Rightarrow -\frac{1}{a^2}=-2b\Rightarrow \frac{1}{2a^2}=b$.

The tangent line at $x=a$ has the equation $(x-a)\,f'(a)=(y-1/a)$. The line intersects g at $x=b$, so the following equation is satisfied: $(b-a)\,f'(a)=(-b^2-1/a)$. Plugging in $b=\frac{1}{2a^2}$ to get

$\left(\frac{1}{2a^2}-a\right)\left(-\frac{1}{a^2}\right)=\left(-\frac{1}{4a^4}-\frac{1}{a}\right)$. Solving for a, we get

$-\left(\frac{1}{2a^4}-\frac{1}{a}\right)=-\left(\frac{1}{4a^4}+\frac{1}{a}\right)\Rightarrow \frac{1}{2a^4}-\frac{1}{a}=\frac{1}{4a^4}+\frac{1}{a}\Rightarrow \frac{1}{4a^4}=\frac{2}{a}\Rightarrow a=\frac{1}{2}$. So $b=\frac{1}{2(1/4)}=2$

$a+b=5/2$

27. $f'(x)=3(x-3)^2(x-2)+(x-3)^3$. So $f'(5)=36+8=44$

28. $\int_{-2}^{2}|x+2|\,dx=\int_{-2}^{2}x+2dx=\frac{1}{2}x^2+2x\Big|_{-2}^{2}=8$

29. Differentiability implies continuity, which implies integrability.

30. $f'(x)>0$, so f is always increasing. Since the graph of f^{-1} is the reflection of f about the line $y=x$, f^{-1} is always increasing.

So it follows that $\int_{1}^{3}f^{-1}(x)\,dx$ is the area beneath $f^{-1}(x)$ from $x=1$ to $x=3$. So it is also the area of the region shaded in the graph at the right. So we can find the area beneath $f(x)$ from $x=0$ to $x=1$, and subtract it from the rectangle formed by the $x-$axis, $y-$axis, $x=1$ and $y=3$.

So $\int_{1}^{3}f^{-1}(x)\,dx=3-\int_{0}^{1}x^5+x+1dx=3-\left(\frac{1}{6}+\frac{1}{2}+1\right)=\frac{4}{3}$