National MA@2008

Solutions

0.	$Area = \frac{1}{2}ab\sin C = \frac{1}{2} \cdot 12 \cdot 18 \cdot \sin(60) = 54\sqrt{3}$
1.	The given equation can be written $2x^2 - x = 3 \Rightarrow (2x - 3)(x + 1) = 0 \Rightarrow x = \frac{3}{2}$ or -1 $\left \frac{3}{2} - (-1)\right = \frac{5}{2}$
2.	The line containing (-2,3) and (3,5) has a slope of 2/5; therefore, a line perpendicular to it has a slope of $-5/2$ and its equation is $y = -\frac{5}{2}x + b$. If the line contains the point (2,0), then $0 = -\frac{5}{2}(2) + b$ and $b = 5$ or (0,5).
3.	$\frac{4-3i}{2-i} \bullet \frac{2-i}{2-i} = \frac{5-10i}{5} = 1-2i \implies a=1, b=-2 \text{ and } 1 + -2 =3$
	2+1 $2-1$ 3
4.	The graph will not intersect the x-axis if the zeros of the function are imaginary. $b^2 - 4ac < 0 \implies 4k^2 - 4k < 0 \implies 4k(k-1) < 0 \implies 0 < k < 1$
4. 5.	The graph will not intersect the x-axis if the zeros of the function are imaginary. $b^{2} - 4ac < 0 \implies 4k^{2} - 4k < 0 \implies 4k(k-1) < 0 \implies 0 < k < 1$ $y = \frac{kx}{z^{2}} \implies k = \frac{yz^{2}}{x} \implies \frac{(2)(4)^{2}}{3} = \frac{y(2)^{2}}{9} \implies y = 24$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1 \implies a^2 = 25 \text{ and } b^2 = 9, \text{ so } c^2 = 16 \text{ and } c = 4$$

The coordinates of the foci are (4,0) and (-4,0), so the length of the rectangle is 8.

Also, if x = 4, $\frac{16}{25} + \frac{y^2}{9} = 1 \implies y = \pm \frac{9}{5}$, which means that the width of the rectangle is $\frac{18}{5}$ and the area of the rectangle is therefore $8\left(\frac{18}{5}\right) = \frac{144}{5}$ or $28\frac{4}{5}$ or 28.8

The equation can be rewritten as $\log_5 (x-1)(x-2) = \log_5 (x+6) \implies x^2 - 3x + 2 = x + 6$ 7. $\Rightarrow (x-2)^2 = 8 \Rightarrow x = \mathbf{2} + \mathbf{2}\sqrt{\mathbf{2}} \text{ (reject } 2 - 2\sqrt{2})$

Graphing form for the circle : $(x - 3)^2 + (y + 2)^2 = 25$ When $x = 7 : (7 - 3)^2 + (y + 2)^2 = 25 \Rightarrow (y + 2)^2 = 9 \Rightarrow y + 2 = \pm 3 \Rightarrow y = 1 \text{ or } -5$ 8. The fourth quadrant point is (7, -5). The slope of the line containing the center (3, -2)and (7, -5) is $-\frac{3}{4}$; therefore, the slope of the tangent line is $\frac{4}{3}$. $y + 5 = \frac{4}{3}(x - 7) \Rightarrow y = \frac{4}{3}x - \frac{43}{3}$ 9. Factoring : $\frac{(x - 1)(x^3 + 1)}{(x - 1)(x^2 - x + 1)} = \frac{(x - 1)(x - 1)(x^2 - x + 1)}{(x - 1)(x^2 - x + 1)} = x + 1 \Rightarrow 2008 + 1 = 2009$ 10. Let x = weight of old alloy $\Rightarrow .25x - .25(12) + 1.00(12) = .40x \Rightarrow x = 60$ 11. $\Rightarrow x = -6 \text{ or } -2$ When x = -6, y = 2(-6) + 9 = -3 and when x = -2, y = 2(-2) + 9 = 5; therefore, the points of intersection are (-6, -3) and (-2, 5).

The sum of the x - and y - coordinates is - 6.

12. The sum will be even if both numbers are odd or both numbers are even; therefore, it makes no difference what number is selected on the first draw. However, the second number drawn must have the same parity (odd or even). There are 20 odd and 20 even numbered tickets. After one is drawn, there will be 39 tickets remaining and 19 that have the same parity as the first. The desired probability is then $\frac{19}{39}$.