1. \( C : 64 \)  
\( \frac{dW}{dt} = kW \rightarrow \int \frac{dW}{W} = kdt \rightarrow \ln W = kt + C \rightarrow W = Ce^{kt} \) at \( t = 0, 2 = Ce^0 \cdot C = 2 \), so \( W = 2e^{kt} \). At \( t = 3 \) he has eaten 16 wings, \( 16 = 2e^{3k} \rightarrow e^{3k} = 8 \rightarrow k = \frac{1}{3} \ln 8 = \ln 2 \). The wing eating function is \( W = 2e^{t\ln 2} \), so at \( t = 5, W = 2e^{5\ln 2} = 2e^{\ln 32} = 2 \cdot 32 = 64 \)

2. \( D : \frac{9\sqrt{3}}{2} \)  
If \( y^2 = x - 3 \), then solving for \( y \) will give us an upper-bound of \( y = \sqrt{x - 3} \) and a lower bound of \( y = -\sqrt{x - 3} \). The side of one of the cross-sections will be \( S = \sqrt{x - 3} - (-\sqrt{x - 3}) = 2\sqrt{x - 3} \) and area of an equilateral triangle is \( \frac{s^2\sqrt{3}}{4} \). The volume can be found by \( V = \frac{\sqrt{3}}{4} \int_3^6 (2\sqrt{x - 3})^2 \, dx = \frac{9\sqrt{3}}{2} \).

3. \( C : 5 \)  
If \( g'(c) = \frac{1}{5} \), then \( f'(x) = 5c = 5 \rightarrow f'(x) = 3x^2 + 2 = 5 \) solving for \( x \) gives us \( x = \pm 1 \). By plugging both 1 and -1 into \( f(x) \) we find that only 1 yields a positive value: 5. If \( f(1) = 5 \), then \( D : 5 \).

4. \( B : 300 \)  
If \( a(t) = -32 \rightarrow v(t) = -32t + v_0 = -32t - 20 \rightarrow s(t) = -16t^2 - 20t + s_0 \), then \( 0 = -16t^2 - 20t + 1400 \rightarrow 16t^2 + 20t - 1400 = 4(4t^2 + 5t - 350) = 4(4t - 35)(t + 10) = 0 \). Plugging into the velocity equation gives us \( v(\frac{35}{4}) = -32(\frac{35}{4}) - 20 = -280 - 20 = -300 \)

5. \( A : \pi \)  
Use washer: \( V = \pi \left[ \left( \frac{1}{x} \right)^2 \right] \bigg[ b \rightarrow \infty \bigg] = \pi \lim_{b \rightarrow \infty} \left( \left[ \frac{1}{x^2} \right] \right) = \pi \lim_{b \rightarrow \infty} \left( \frac{1}{b} \right) = \pi \cdot \lim_{b \rightarrow \infty} \left( \frac{1}{b} - (-1) \right) = \pi \)

6. \( C : \frac{2\pi - 3\sqrt{3}}{2} \)  
The inner loop of the graph will occur in between the angles when \( 1 - 2\sin \theta = 0 \). \( 2\sin \theta = 1 \rightarrow \) \( \sin \theta = \frac{1}{2} \). \( \theta = \frac{\pi}{6}, \frac{5\pi}{6} \). \( A = \frac{\frac{5\pi}{6}}{\frac{\pi}{6}} \int_1^{\frac{5\pi}{6}} (1 - 2\sin \theta)^2 \, d\theta = \frac{\frac{5\pi}{6}}{\frac{\pi}{6}} \int_1^{\frac{5\pi}{6}} (1 - 4\sin \theta + 4\sin^2 \theta) \, d\theta = \frac{5\pi}{6} \int_1^{\frac{5\pi}{6}} \left( 1 - 4\sin \theta + 4\cdot\frac{1 - \cos 2\theta}{2} \right) \, d\theta = \frac{5\pi}{6} \int_1^{\frac{5\pi}{6}} \left( 3\theta + 4\cos \theta - 2\sin 2\theta \right) \, d\theta = \frac{2\pi - 3\sqrt{3}}{2} \)

7. \( B : 29 \)  
\( \int_0^6 (t^2 + 7t - 4) \, dt = \left[ \frac{t^3}{3} + \frac{7t^2}{2} - 4t \right]_0^6 = 72 + 126 - 24 = 29 \)
8. \[ C : \frac{14\pi e^{\frac{3}{2}}}{3} \]  
Use shells- \[ 2\pi \int_0^{\frac{\sqrt{2}}{3}} \sqrt{x^2 + e^x} \, dx \]
\[ u = x^2 + e, \, du = 2xdx \rightarrow xdx = \frac{du}{2} \]
\[ \frac{\sqrt{\pi}}{3} \int_e^4 \sqrt{u} \, du = \pi \left[ \frac{2}{5} \right] \frac{e^{\frac{3}{2}}}{e} = \frac{2\pi}{3} \frac{\left(8e^{\frac{3}{2}} - e^{\frac{3}{2}}\right)}{3} = \frac{14\pi e^{\frac{3}{2}}}{3} \]

9. \[ A \cup B \cup C \]
If \[ A : (0,1) \cup (2,\infty) \], then \( v(t) = 3t^2 - 6t \). Velocity will be negative on the interval \((0,2)\) and positive \((2,\infty)\). \( a(t) = 6x - 6 \) and changes from negative to positive at \( t = 1 \). Speed is increasing when velocity and acceleration have the same sign. Velocity and acceleration are both negative from \((0,1)\) and both positive from 
\((2,\infty)\).

10. \[ D : \pm \frac{4}{15} \]
The maximum possible error diameter within the acceptable parameters will occur when the maximum error occurs in the opposite direction for the height. If the height has a negative error, the diameter may have a larger error and still meet the volume requirements. \( V_{cone} = \frac{1}{3} \pi r^2h \rightarrow dV = \frac{2}{3} \pi rhdr + \frac{1}{3} \pi r^2dh \). The maximum positive error in volume, \( \frac{\pi}{6} \), and maximum negative error in height, \( -\frac{2}{3} \), will give us the maximum possible error in diameter. \( dV = \frac{2}{3} \pi rhdr + \frac{1}{3} \pi r^2dh \rightarrow \frac{\pi}{6} = \frac{2}{3} \pi \left(3\frac{1}{2}\right) (5) dr + \frac{1}{3} \pi \left(\frac{3}{2}\right) \left(-\frac{2}{3}\right) \). Solving for \( dr \), we find \( dr = \frac{2}{15} \)
therefore the error in diameter will be \( \pm \frac{4}{15} \) depending on which values are used for \( dv \) and \( dh \).

11. \[ B \left[ 1, -\frac{3}{5} \right] \]
Area of the region is \( \int_0^1 (2x - x^2)dx = \frac{4}{3} \)
\[ x = \frac{1}{A} \int_a^b x[f(x) - g(x)]dx = \frac{3}{4} \int_0^1 x[2x - x^2]dx = \frac{3}{4} \left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^1 = \frac{1}{1} \]
\[ y = \frac{1}{2A} \int_a^b [(f(x))^2 - (g(x))^2]dx = \frac{3}{8} \left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^1 = -\frac{3}{5} \]

12. \[ E : 2\sqrt{10} \]
Total acceleration will be \( \sqrt{\frac{d^2x}{dt^2} + \frac{d^2y}{dt^2}} = \sqrt{(3t)^2 + \left(-\frac{8}{t^2}\right)^2} \). At \( t = 2 \) \( \sqrt{(6)^2 + (-2)^2} = 2\sqrt{10} \)

13. \[ C : \frac{7}{4} \]
The distance between the foci is \( 2c \) from the equation \( a^2 - b^2 = c^2 \rightarrow 2a \frac{da}{dt} - 2b \frac{db}{dt} = 2c \frac{dc}{dt} \) and the rate of change of the distance will be \( \frac{dc}{dt} \). If \( \frac{x^2}{25} + \frac{y^2}{9} = 1 \), then \( a = 5, b= 3 \) and \( c \) will equal 4. \( a \) and \( b \) will decrease at half the rate of their respective axes. \( \frac{da}{dt} - \frac{db}{dt} = \frac{dc}{dt} \rightarrow 5 \left(\frac{1}{2}\right) - 3 \left(-\frac{1}{3}\right) = 4 \frac{dc}{dt} \). Solving for \( \frac{dc}{dt} = \frac{7}{8} \), so the rate of change is \( \frac{7}{4} \).
14. Use shells - \( V = 2\pi \int_{0}^{4} (6-x) \left( 2-\sqrt{x} \right) dx = 2\pi \left[ 12x - x^2 - 2x^{3/2} + \frac{2}{5} x^{5/2} \right]_{0}^{4} = \frac{128\pi}{5} \)

15. **B: MVT for D** The MVT for Derivatives states that for a continuous, differentiable function over a given interval there must be at least one point in between the endpoints that the rate of change at that point must be equal to average rate between the endpoints. In this case the care goes 30 miles in .5 hours giving an average rate of change of 60 mph.

16. \( y(x) = e^x \) if \( y'(e^3) = e^5 \), then \( e^x = Ce^{\frac{(ln x)^2}{2}} \), \( C = e^\frac{9}{2} \), \( x = e^\frac{1}{2} \), \( y = e^\frac{1}{2} \cdot e^{\frac{9}{2}} = e^{\frac{11}{2}} \)

17. \( C : 2\pi \int_{0}^{b} x \cdot \sqrt{1 + x^2} \ dx \) As we are revolving about the y-axis, the radius will be \( x \) and \( \frac{d}{dx} \left[ \frac{x^2 + 1}{2} \right] = x \)

18. \( b \left( \frac{1 + 6x - 3x^2}{b} \right) \) → \( b \left( \frac{3x^2 - x^3}{b} \right) = 3 \) → \( b + 3b^2 - b^3 = 3b \) → \( b^3 - 3b^2 + 2b = 0 \)

19. **D: 71/6** The curves intersect at \( x = -3, 0, 1 \) \( A = \int_{-3}^{0} (x^3 + 2x^2 - 3x) \ dx \) + \( \int_{0}^{1} (3x - (x^3 + 2x^2)) \ dx = \frac{71}{6} \)

20. **B: 6\sqrt{3}** The area function for the triangle will be \( A(x) = \frac{1}{2} (2x)(9-x^2) = 9x - x^3 \) \( A'(x) = 9 - 3x^2 = 0 \)

21. \( A : \frac{\pi}{6} \)

22. **E: \( y - \frac{\pi}{3} = -4(x - \sqrt{3}) \)** \( y = \arctan x \) \( y'(x) = \frac{1}{1+x^2} \) \( m = \frac{1}{4} \), \( m_\perp = -4 \). For \( x = \sqrt{3} \), \( y = \frac{\pi}{3} \). The normal line is \( y - \frac{\pi}{3} = -4(x - \sqrt{3}) \)

23. **B: 19600** \( F = mg = 10 kg \cdot 9.8 m/s^2 = 98 \) Newtons \( W = \int_{0}^{20} F(x) \ dx = \int_{0}^{20} 98x \ dx = [\frac{49x^2}{2}]_{0}^{20} = 19600 \) joules
24. \[ D: \frac{7}{3} \] Max slope will be 2\textsuperscript{nd} der. set equal to 0. 
\[ f'(x) = -3x^2 + 4x + 1 \rightarrow f''(x) = -6x + 4 = 0 \]

\[ x = \frac{2}{3} \]

\[ f''(x) \] is negative for all \( x \) so \( x = \frac{2}{3} \) will be a maximum for slope. 
\[ f'(\frac{2}{3}) = -3\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right) + 1 = \frac{7}{3} \]

25. \[ D: 180\pi^2 \] Pappus \( \rightarrow V = 2\pi R \left( \text{Area} \right) \) The equation is a circle with radius of 3 and center (4,-5) so the area of the region bounded is \( 9\pi \). The distance between can be found by find the distance from the center of the circle and the point of intersection of the line and it’s normal that includes the center. The line is \( 4x - 3y = 19 \) and the normal is \( 3x + 4y = -8 \). Solving the system gives us the point (-4,1). The distance between (-4,1) and (4,-5) is 10. The volume of the solid will be \( 180\pi^2 \).

26. \[ C: \left[ 8\sqrt{3}, -\frac{18}{\pi^2} \right] \] \( i''(t) = 2\sec^2 t \tan t \), \( j''(t) = -\frac{2}{t^2} \), \( i''\left(\frac{\pi}{3}\right) = 2\sec^2\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{3}\right) = 8\sqrt{3} \)

\[ j''\left(\frac{\pi}{3}\right) = -\frac{2}{\left(\frac{\pi}{3}\right)^2} = -\frac{18}{\pi^2} \]

So the vector will be \( \left( 8\sqrt{3}, -\frac{18}{\pi^2} \right) \)

27. \[ A: 45^\circ \] Setting \( f(x) \) and \( g(x) \) equal we find that they intersect at \( x = 3, \frac{4}{3} \). We want the greatest value of \( x \) so we use \( x = 3 \). The slope of \( f(x) \) at 3 is \( f''(3) = 4(3) - 10 = 2 \) and slope of \( g(x) \) is \( g''(3) = -2(3) + 3 = -3 \). The tangent of the angle between two slopes can be found with \( \tan \theta = \frac{m_1 - m_2}{1 + m_1m_2} \).

\[ \tan \theta = \frac{-2 - (-3)}{1 + (-2)(-3)} = \frac{5}{5} = 1 \]

\[ \arctan (-1) = 135^\circ \]

So the smaller angle would be \( 45^\circ \)

28. \[ C: 216 \] \( V = \pi r^2 h \rightarrow 36 \chi = \chi r^2 h \rightarrow h = \frac{36}{r^2} \)

\( \text{SA} = 2\pi r^2 + 2\pi rh \rightarrow \)

\( \rightarrow \text{Weight} = 2\chi r^2 \left( \frac{9}{\chi} \right) + 2\chi rh \left( \frac{4}{\chi} \right) \rightarrow r^2 \cdot 9 + 2r \left( \frac{36}{r^2} \right) \cdot 4 = 18r^2 + \frac{288}{r} \). Min. weight \( W' = 36r - \frac{288}{r^2} = 0 \)

Solving for \( r \) we get \( r = 2 \). Plugging into the weight function we get \( W(2) = 18(2)^2 + \frac{288}{2} = 216 \)

29. \[ B: \frac{1}{2} \ln (2 + \sqrt{3}) \] Arclength = \( \int_a^b \sqrt{\left( \frac{dy}{dt} \right)^2 + \left( \frac{dx}{dt} \right)^2} \, dt = \frac{1}{2} \int_0^{\frac{\pi}{2}} \sec \theta \, dt = \frac{1}{2} \left[ \ln (\sec \theta + \tan \theta) \right]_0^{\frac{\pi}{2}} \)

\( \rightarrow \int_0^{\frac{\pi}{2}} \left( -\tan \theta/2 \right)^2 + \left( 1/2 \right)^2 \, dt = \frac{1}{2} \int_0^{\frac{\pi}{2}} \sqrt{\tan^2 \theta + 1} \, dt = \frac{1}{2} \int_0^{\frac{\pi}{2}} \sec \theta \, dt = \frac{1}{2} \left[ \ln (\sec \theta + \tan \theta) \right]_0^{\frac{\pi}{2}} = \frac{1}{2} \ln (2 + \sqrt{3}) \)

30. \[ C: 34 \] Marginal cost will be the derivative of the cost function \( C(x) = x \cdot p(x) - r(x) \) where \( C(x) \) is the cost function, \( r(x) \) is the profit function and \( p(x) \) is the price per Snitch. 

\( C(x) = x \cdot \frac{x^2 - 4x + 9}{2} - \left( \frac{1}{2}x + 4 \right) \)
\[
\frac{1}{2}(x^3 - 4x^2 + 9x) - \left(\frac{1}{2}x + 4\right)
\]
\[
C'(x) = \frac{1}{2}(3x^2 - 8x + 9) - \left(\frac{1}{2}\right) = \frac{3}{2}x^2 - 4x + 4
\]
\[
C'(6) = \frac{3}{2} \cdot 6^2 - 4 \cdot 6 + 4 = 34
\]

Answers:

1. C
2. D
3. C
4. B
5. A
6. C
7. B
8. C
9. A
10. D
11. B
12. E
13. C
14. D
15. B
16. A
17. C
18. C
19. D
20. B
21. A
22. E
23. B
24. D
25. D
26. C
27. A
28. C
29. B
30. C