1. Find the area formed between the two curves $y = x^3 + x + 8$ and $y = 3x^2 + 7x$

A. 0 B. $\frac{81}{4}$ C. $\frac{81}{2}$ D. 81 E. NOTA

2. What is the area of the smallest circle that circumscribes a triangle with side lengths 3, 4 and 5?

A. $\frac{9\pi}{4}$ B. 4π C. $\frac{25\pi}{4}$ D. 12 E. NOTA

3. A rectangle CAKE is inscribed in the functions $f(x) = x^2 - 4$ and $g(x) = 4 - x^2$ where vertices C and A lie on f(x) and vertices K and E lie on g(x). Find the area of the largest such rectangle CAKE.

A.
$$\frac{64\sqrt{6}}{9}$$
 B. $64\sqrt{3}$ C. $\frac{16\sqrt{3}}{9}$ D. $\frac{64\sqrt{3}}{9}$ E. NOTA

4. Consider the shaded region R that is formed by a larger semicircle with diameter of one unit, and two smaller circles whose diameters sum to one unit. Maximize the area of R.

A.
$$\pi$$
 B. $\frac{\pi}{2}$ C. $\frac{\pi}{4}$ D. $\frac{\pi}{8}$ E. NOTA

5. The diagonals of quadrilateral LEAK have measure LA = 12 and EK = 8. Find the largest area LEAK can be under these conditions.

A. $96\sqrt{2}$ B. 96 C. $48\sqrt{2}$ D. 48 E. NOTA.

6. For a cube with side length 3 m, find the largest tetrahedron that will fit inside of the cube.

A. 13.5 B. 9 C. 4.5 D. 2.25 E. NOTA

7. Gabriel's horn is determined by the revolution of the function $f(x) = \frac{1}{x}$ about the x-axis for $x \ge 1$. What is the volume determined by Gabriel's horn and the plane x = 1?

A. 1 B.
$$\pi$$
 C. $\frac{\pi^2}{2}$ D. ∞ E. NOTA



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NOTA=None of the Above

8. Archimedean solids are the convex polyhedra that have a similar arrangement of nonintersecting regular convex polygons of two or more different types arranged in the same way about each vertex with all sides the same length. An example of one is the truncated icosahedron or a soccer ball. How many Archimedean solids are there?

A. 5 B. 12 C. 13 D. 20 E. NOTA

9. Consider a solid whose base is determined by the two functions sin(x) and -sin(x) on the interval $[0,\pi]$. The solid is formed by cross-sections taken perpendicular to the x-axis in the shape of equilateral triangles. Determine the volume of this solid.

A. $\frac{\pi\sqrt{3}}{4}$ B. $\frac{\pi\sqrt{3}}{2}$ C. $\pi\sqrt{3}$ D. $\left(\frac{\pi+1}{2}\right)\cdot\sqrt{3}$ E. NOTA

10. There are only two right triangles with integral side lengths that have an area equal to 210. Only one of these triangles has two sides that differ by one unit. Find the largest side of that triangle. A. 26 B. 27 C. 28 D. 29 E. NOTA

11. Find the volume of a sphere with Surface area 400π .

A. $\frac{4000\pi}{3}$ B. 1000π C. $\frac{6400\pi}{3}$ D. 1200π E. NOTA

12. Allow for an ellipse of equation $\frac{(x-5)^2}{16} + \frac{(y-8)^2}{4} = 1$ to be completely revolved around the axis 3x + 4y = 7 so as to create a torus. Find the volume, in cubic units, contained in that torus. A. 128π B. 180π C. 224π D. 256π E. NOTA

13. A sports field in a city park has an irregular shape. Measurements taken 20 yds apart reveal widths of 40, 45, 50, 45 and 35 yards. Using Simpson's rule, approximate the area of the field in square yards.

A. $\frac{10700}{3}$ B. $\frac{5350}{3}$ C. $\frac{9100}{3}$ D. $\frac{4550}{3}$ E. NOTA

14. Find the area of one leaf in the rose $r = 4\cos(2\theta)$ in square units.

A. π B. 2π C. 3π D. 4π E. NOTA

Mu Area and Volume – National MAO 2008NOTA=None of the Above15. The lines y = 3x + 1, x + y = 5 and y = 1 form a triangle. What is the area of that triangle in square units?A. 3B. 6C. 12D. 24E. NOTA

16. The interior shape of Linas' favorite cereal bowl can be approximated by rotating the function $L(x) = \frac{x^2}{4}$ from [0,2] about the y-axis. The answer to number 16 is the volume of the milk in cubic units that can fill his bowl, assuming it would spill over if any milk exceeded the bowl's lip or edge.

A. $\frac{\pi}{2}$ B. π C. 2π D. $2\pi^2$ E. NOTA

17. The pentagon TEBOW is composed of an equilateral triangle and a square so that all sides have length x feet. If the area of TEBOW is 3000 square feet, what is the value of x in feet?

A. $20 - 5\sqrt{13}$ B. $30\sqrt{13} + 15$ C. $80 - 20\sqrt{3}$ D. $10\sqrt{30}$ E. NOTA

18. Triangle IBC's is a mutable triangle with turning angles I and B, while C remains a fixed right angle and side IC also remains the same length. Initially, IBC has area $12\sqrt{3}$ units² with I at 30° but it is growing at 1 radian/sec. Find the rate at which the area is increasing in units²/sec when the angle B is 30°.

A. 144 B. $4\sqrt{3}$ C. $2\sqrt{6} + 8$ D. $16\sqrt{2}$ E. NOTA

19. For a right circular cone, the lateral area is 575π and the base area is 529π . Find the volume of the cone.

A. 1104π B. 1215π C. 1789π D. $\frac{2116\pi\sqrt{6}}{3}$ E. NOTA

For Problems 20 and 21, three points A=(1,3,4), B=(2,-4,5), C=(-3, 2, 2) and the origin D form the four vertices of a convex tetrahedron ABCD in space.

20. Find the area of the triangle CAB.

A. 15 B. $\sqrt{941}$ C. $\sqrt{946}$ D. $\sqrt{1070}$ E. NOTA

21. Find the volume of the convex tetrahedron ABCD.

A. 107 B. 33 C. 37√37 D. 105 E. NOTA

22. Rotate the area determined by x-axis, the function $f(x) = \frac{1}{x^2 + 4}$ and the horizontal lines x = 0 & x = 3around the y-axis. The volume of this jelly-shaped dome in cubic units is the answer to number 22. A. $\pi \ln\left(\frac{13}{4}\right)$ B. $\frac{T \operatorname{an}^{-1}(3.25)}{2}$ C. $\ln\left(\frac{13}{4}\right)$ D. $\frac{\pi \cdot T \operatorname{an}^{-1}(3.25)}{2}$ E. NOTA 23. Maximize the integral for x given constant c: $\int_{c}^{x} \sin(t) dt$ B. 2π D. 2 E. NOTA Α.π 24. What is the minimum volume of a circular cone circumscribing a sphere of volume 36π ? A. $\frac{243\pi}{4}$ C. 72π B. 81π D. 64π E. NOTA 25. Determine the sum of the perimeters of all right triangles with integral side lengths where one of the sides has a length of 5 units. D. 60 A. 30. B. 42 C. 49 E. NOTA 26. Find the lateral area of a right square pyramid with base area 64 in² and volume 48 in³. B. $4\sqrt{337}$ C. $8\sqrt{145}$ D. $\frac{64\sqrt{3}}{3}$ E. NOTA A. 36 27. What is the area formed by the triangles with vertices (2, 4), (3, 7), and (-2, 8) in square units? A. 4 B. 8 C. 16 D. 32 E. NOTA

28. Using shell method, which integral will satisfy the volume formed by rotating the function $y = \frac{1}{x}$ along the interval x = 2 to x = 9 about the x-axis.

A.
$$\pi \int_{2}^{9} \frac{dx}{x^{2}}$$
 B. $\pi \int_{\frac{1}{2}}^{\frac{1}{2}} (y - y^{2}) dy$ C. $2\pi \int_{\frac{1}{2}}^{\frac{1}{2}} (2y - 1) dy$ D. $2\pi \int_{\frac{1}{2}}^{\frac{1}{2}} (1 - 2y) dy$ E. NOTA

29. Evaluate the correct integral solution to number 28.

A. $\frac{7\pi}{18}$ B. $\frac{7\pi}{36}$ C. $\frac{49\pi}{81}$ D. $\frac{49\pi}{162}$ E. NOTA

30. A right trapezoid (two angles are right) has sides with lengths 3, 4, 5 and 6. Find the only possible area of that trapezoid.

A. 13.5 B. 15 C. 16 D. 18 E. NOTA