For all questions, choice E) NOTA denotes **N**one **O**f **T**he **A**bove are correct.

1) Evaluate: $\lim_{x\to\infty} \frac{9x^3 - 3x^2 + 7x - 2007}{3x^3 + 6x + 2007}$

A) –1 B) 0 C) 1 D) 3 E) NOTA

2) Given: $f(x) = 9x^2$ Evaluate: $\lim_{t\to 0} \frac{f(t+1) - f(1)}{t}$

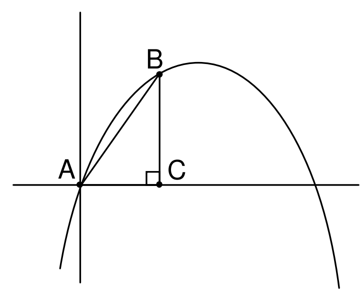
A) 0 B) 9 C) 18 D) 27 E) NOTA

3) Find the second derivative of $y = \frac{e^{x^2}}{2x}$ with respect to x; $x \neq 0$.

A) $y'' = \frac{e^{x^2}(2x^2+1)}{2x^3}$ B) $y'' = \frac{e^{x^2}(2x^2-1)}{2x^3}$

C) $y'' = \frac{e^{x^2}(x^4-2x^2+1)}{x^3}$ D) $y'' = \frac{e^{x^2}(2x^4-x^2+1)}{x^3}$ E) NOTA

4) A right triangle is constructed in the Cartesian plane with vertices A, B, C. A is the origin (0,0), B is located in the first quadrant on the graph $y = 2x - x^2$, and C is a point on the x-axis with the same x-coordinate as B. Angle ACB is a right angle. Find the area of the triangle that has maximum area.



A) $\frac{16}{27}$ B) $\frac{4}{3}$ C) $\frac{32}{27}$ D) $\frac{8}{3}$

E) NOTA

5) Let $h(x) = \sqrt{x + \sqrt{x + \sqrt{x + \ldots}}}$, $h(x) \geq 0$ Find $h'(2)$

A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{3}{4}$ E) NOTA

6) Let $h(x) = \begin{cases} 9 - x^2, & x < 1 \\ a & x = 1 \\ x + 7 & x > 1 \end{cases}$

Which of the following describes the choice for the real number a so that $\lim_{x\to 1} h(x)$ exists?

A) a must be equal to 8 for the limit to exist.
B) There is no choice for a that will make the limit exist.
C) a must be a positive real number for the limit to exist.
D) a can be any real number for the limit to exist.
E) NOTA

7) Evaluate $\frac{dy}{dx}$ of the curve $4x^3 - x^2y^2 + 2y^2 = 0$ at the point $(-1,2)$.

A) $-\frac{5}{2}$ B) 0 C) 5 D) 8 E) NOTA

8) Over the domain $(1,4)$ the function $y = x^3 - 2x^2 + x - 1$ has which of the following properties?

I) Concave up II) Concave down III) Increasing IV) Decreasing

A) I only B) IV only C) I & III only D) II & III only E) NOTA

9) Given that the limit $\lim_{n\to\infty} a^n n^a$ converges to a finite value for some value of *a*, how many of the following CANNOT be equal to *a*?

I) –2 II) –1 III) $-\frac{1}{2}$ IV) $\frac{1}{2}$ V) 1 VI) 2

A) 1 B) 2 C) 3 D) 4 E) NOTA

10) Find the minimum distance from the curve $x = y^2 + 3$ to the point $(2,-5)$

A) 2 B) $2\sqrt{5}$ C) $\sqrt{34}$ D) $2\sqrt{10}$ E) NOTA

11) The volume of a sphere is increasing at a constant rate of 4π cubic meters per minute. Find the rate of change of the surface area of the sphere, in square meters per minute, when the radius is 2 meters long.

A) 2π B) 4π C) 8π D) 16π E) NOTA

12) Evaluate: $\lim_{n\to\infty}\left(1 - \frac{1}{n}\right)^{2n-3}$.

A) $e^{-2} - 3$ B) e^{-2} C) $e^2 - 3$ D) e^2 E) NOTA

13) Find the maximum value of the function $h(x) = x^{1/x}$ for $x > 0$.

A) 1 B) $\sqrt{2}$ C) $\sqrt[3]{3}$ D) ∞ E) NOTA

14) Let *n* be a positive integer. In terms of *n* find the coefficient of $\frac{d^n}{dx^n}\left[x^{n!}\right]$.

A) $\frac{(n!)!}{n!((n-1)!)!}$ B) $\frac{(n!)!}{(n!-n-1)!}$ C) $\frac{(n!)!}{(n!-n)!}$ D) $\frac{(n!)!}{(n!)!-(n-1)!}$

E) NOTA

15) $\lim_{x\to 3}\left(\frac{x}{3}+\pi-1\right)=\pi$. The $\varepsilon-\delta$ definition of the limit requires that for each $\varepsilon>0$, there must exist a $\delta>0$ for the limit to exist. Which of the following is the largest possible choice for δ to satisfy the definition when $\varepsilon=0.3$?

A) $\frac{0.3}{\pi}$ B) 0.1 C) 0.3 D) 0.9 E) NOTA

16) Find the slope of the polar graph $r=1-\cos(2\theta)$ when $\theta=\frac{\pi}{3}$.

A) $-3\sqrt{3}$ B) $\frac{\sqrt{3}}{5}$ C) $\frac{3\sqrt{3}}{5}$ D) $\sqrt{3}$ E) NOTA

17) Let $g(x)$ be a continuous and differentiable function defined for all real numbers. The table shows the value of $g(x)$ and its derivative $g'(x)$ for certain values of *x*.

x	–2	–1	0	1	2	3
$g(x)$	–6	–2	0	2	0	–2
$g'(x)$	2	1	0	0	1	2

Which of the following MUST be true regarding the function $g(x)$?

I) $g(x)$ is strictly increasing.

II) $g(x)$ has a local minimum at $x = 1$.

III) $g(x)$ has a critical point at $x = 0$.

IV) $g(x)$ is concave up on the interval $(1,3)$.

A) I only B) III only C) II & III only D) I, III, & IV only E) NOTA

18) Evaluate: $\lim_{n\to\infty}\frac{1}{n}\left[e^{1/n}+e^{2/n}+e^{3/n}+\ldots+e^{n/n}\right]$

A) $e-1$ B) e C) $e+1$ D) Does not exist E) NOTA

19) Given a continuous and differentiable function, *f*, with $f(0)=1$, $f'(0)=0$, $f''(0)=-1$, and $g(x)=(1+f(x))^2$. Calculate $g''(0)$.

A) –4 B) –2 C) 0 D) 2 E) NOTA

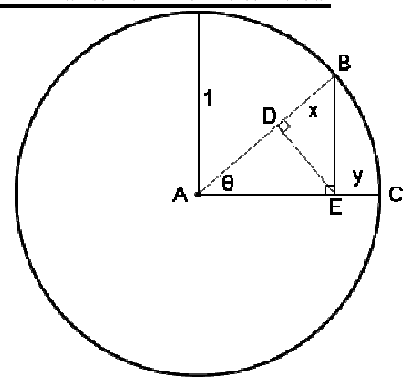
20) Given $F(x)=\int_0^x \sin(t^2)\,dt$. Over which interval is $F(x)$ NOT concave down?

A) $\left(-\sqrt{\frac{\pi}{2}},-\sqrt{\frac{\pi}{4}}\right)$ B) $\left(-\sqrt{\frac{\pi}{4}},0\right)$ C) $\left(\sqrt{\frac{\pi}{4}},\sqrt{\frac{\pi}{2}}\right)$ D) $\left(\sqrt{\frac{\pi}{2}},\sqrt{\frac{3\pi}{4}}\right)$

E) NOTA

21) In the figure the lengths of AB and AC are both equal to 1, the length of BD is denoted as x and the length of CE is denoted as y. BE is perpendicular to AC and DE is perpendicular to AB. Evaluate: $\lim\limits_{\theta \to 0} \frac{x}{y}$.

NOTE: Figure is not drawn to scale.



A) 0 B) 1 C) 2
D) Does not exist E) NOTA

22) Evaluate: $\lim\limits_{n \to \infty} \frac{1+2^n+3^n+4^n}{5^n}$

A) 1 B) 2 C) 10 D) Does not exist E) NOTA

23) Find the sum of the value(s) for c that satisfy the Mean Value Theorem for Derivatives of the function $f(x) = x^3 - x$ over the domain $[-2,1]$.

A) –2 B) –1 C) 0 D) 1 E) NOTA

24) Evaluate: $\lim\limits_{x \to -1} \frac{2x^2-3x+1}{x+1}$

A) –7 B) 0 C) 6 D) Does not exist E) NOTA

25) Using differentials approximate $\sqrt[3]{997}$ given that $10^3 = 1000$.

A) $\frac{997}{100}$ B) $\frac{499}{50}$ C) $\frac{999}{100}$ D) $\frac{2999}{300}$ E) NOTA

26) Let there exist a real function $f(x)$ such that $\lim\limits_{x \to c} f(x)$ does not exist for some real number c, but $\lim\limits_{x \to c} |f(x)| = L$ exists for some positive real number L. How many of the following MUST be true?

I) $\lim\limits_{x \to c^+} f(x)$ and $\lim\limits_{x \to c^-} f(x)$ both exist

II) $\lim\limits_{x \to c} |f(x) - L| = 0$

III) $f(c)$ does not exist

IV) $\lim\limits_{\Delta x \to 0} \frac{|f(c+\Delta x) - f(c)|}{\Delta x}$ exists

A) 1 B) 2 C) 3 D) 4 E) NOTA

27) If $\lim_{x\to\infty} \sqrt{Ax^2+12x} - Bx = 3$, find $A+B$, given $A, B > 0$.

A) 4 B) 6 C) 12 D) Does not exist E) NOTA

28) Given $f(x) = x^3 + x + 1$ and $g(x) = f^{-1}(x)$. Find $g'(3)$.

A) $\frac{1}{28}$ B) $\frac{1}{4}$ C) 4 D) 28 E) NOTA

29) Which of the following functions are differentiable at every value of x on the domain of all real numbers?

I) $y = x \cdot |x|$ II) $y = (x-1)\cdot|x|$ III) $y = |x-1|^3$ IV) $y = x^3 \cdot |x|$

A) II only B) I & IV only C) I, III & IV only D) All are differentiable
E) NOTA

30) Evaluate: $\lim_{x\to 0} \frac{x\sin(x)}{e^{x^2}-1}$

A) –1 B) 0 C) $\frac{1}{2}$ D) 1 E) NOTA