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1. **[A]** The probability that no girls are selected is the same as the probability that only boys are selected. So divide the number of combinations of three boys by the number of combinations of three students.

$$\frac{{}_4C_3}{{}_6C_3} = \frac{\frac{4!}{3!(4-3)!}}{\frac{6!}{3!(6-3)!}} = \frac{\frac{4\cdot3\cdot2\cdot1}{(3\cdot2\cdot1)(1)}}{\frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{(3\cdot2\cdot1)(3\cdot2\cdot1)}} = \frac{4}{20} = \frac{1}{5}.$$

2. **[C]** $x = \frac{3}{4}$, $y = \frac{4}{4}$, so the hypotenuse of the triangle is $\frac{5}{4}$. $\sec\beta = \frac{1}{\cos\beta} = \frac{h}{x} = \frac{\frac{5}{4}}{\frac{3}{4}} = \frac{5}{3}$.

3. **[D]** *c* is in the first row and the third column, so multiply the first row of the first matrix by the third column of the second matrix. $c = (2)(5) + (3)(0) + (-4)(2) = 10 + 0 - 8 = 2$. Likewise, multiply the third row by the second column to find *j*. $j - (0)(1) + (2)(-1) + (-2)(3) = 0 - 2 - 6 = -8$. $|c - j| = |2 - (-8)| = 10$.

4. **[B]** (I) is true, because π is irrational but real. (II) is false, because the magnitude of the vector is not 1. $\sqrt{(1)^2 + (1)^2} = \sqrt{2}$. (III) is true, because the functions $f(x) = x$ and $f(x) = \sin(x)$ are both odd, and the product of two odd functions is an even function.

5. **[A]** The *y*-intercept is $(-b)^3(a) = -ab^3$. There are two *x*-intercepts: b and $-a$. Note that although *b* is a repeating root, it is only one *x*-intercept. The product is $(-ab^3)(b)(-a) = a^2b^4$.

6. **[A]** Clearly, $T = 1$, because column (1) cannot sum to more than 14. Now, we also know that $(P + N) > 9$. Now, look at column (3). Because $T = 1$ and we know *N* and *P* are even, that means that column (4) must also carry a 1 into column (3). So $(S + P) > 9$. Note that none of $\{N, P, S\}$ can be 2, because 2 cannot add to any of the remaining numbers $\{4, 6, 8\}$ to be more than 9, except with 8 – but 0 is not a possible digit, so the sum of 10 is impossible. By process of elimination, that means that $R = 2$. Looking at column (4) now, *S* and *P* must be 8 and 4, though not necessarily in that order, because that is the only available combination that sums to 12. That means $N = 6$, and then looking at column (1) again, $P = 8$ (it cannot be 4, because *S* cannot be 0.) Finally, $S = 4$, and $|R + S - T| = |2 + 4 - 1| = 5$.

	(1)	(2)	(3)	(4)
	P	*S*	*T*	*S*
+	*N*	*R*	*N*	*P*
T	*S*	*N*	*P*	*R*

7. **[B]** Reducing to the simple form $r = \frac{ep}{1 - e\cos\theta}$, $r = \frac{4}{2 - \cos\theta} = \frac{4}{2\left(1 - \frac{1}{2}\cos\theta\right)} = \frac{2}{1 - 0.5\cos\theta} = \frac{(0.5)(4)}{1 - 0.5\cos\theta}$.

Since the eccentricity is between 0 and 1, the conic is an ellipse.

8. [B] The function's value is greatest when the denominator is smallest.

$P(4,40)=\left|\ \frac{40}{10}-\log(400)\ \right|=\left|\,4-\left(2\log 2+\log 100\right)\right|=\left|4-2.6\right|=1.6$

$P(5,32)=\left|\ \frac{32}{10}-\log(500)\ \right|=\left|\,3.2-\left(\log 1000-\log 2\right)\right|=\left|3.2-2.7\right|=0.5$

$P(8,20)=\left|\ \frac{20}{10}-\log(800)\ \right|=\left|\,2-\left(3\log 2+\log 100\right)\right|=\left|2-2.8\right|=0.8$

$P(10,12)=\left|\ \frac{12}{10}-\log(1000)\ \right|=\left|\,1.2-\left(\log 1000\right)\right|=\left|1.2-3\right|=1.8$

9. [B] Use the law of cosines. $x^2=9^2+5^2-2(9)(5)\left(\frac{16}{25}\right)=106-\frac{288}{5}=106-57.6=48.4$.

This is very close to 7^2.

10. [D] The longer diagonal of a regular hexagon with one side *x* measures $2x$. A shorter diagonal measures $x\sqrt{3}$. So according to the problem $(2x)^2-\left(x\sqrt{3}\right)^2=64$, and $4x^2-3x^2=64\ \rightarrow\ x^2=64$. Since the area of a regular hexagon is $\frac{3}{2}x^2\sqrt{3}$, the area of this hexagon is $96\sqrt{3}$.

11. [A] The distance is along a perpendicular line segment. Consider the perpendicular line $y=-\frac{1}{2}x$. It intersects the lines at (0,0) and $(-2,1)$. The distance between these points is $\sqrt{5}$.

12. [D] $\left(\frac{9}{10}\right)\left(\frac{21}{20}\right)\left(\frac{4}{5}\right)Q=\frac{189}{250}Q$.

13. [A] By Heron's formula, the triangle's area is $\sqrt{9(9-4)(9-6)(9-8)}=3\sqrt{15}$. Using the area formula on the longest side (which has the shortest altitude) $A=\frac{1}{2}bh\ \rightarrow\ 3\sqrt{15}=4h\ \rightarrow h=\frac{3\sqrt{15}}{4}$. $h^2=\frac{135}{16}$.

14. [B] Use the rational root theorem with synthetic division to find that the roots are $\{-2,3,4\}$.

15. [B] The domain of $\text{Cos}^{-1}(x)$ is $[-1,1]$. Since the range is $[0,\pi]$, any number in the range has a real square root. Because $\sin(x)\geq 1$ for all *x*, the radicand of the second root always has a real square root. So the final domain of the function is $[-1,1]$, and half of that is positive.

16. [C] Dividing the interval into seven parts, the domain that she is graphing is $\left\{\frac{2\pi}{7},\frac{4\pi}{7},\frac{6\pi}{7},\frac{8\pi}{7},\frac{10\pi}{7},\frac{12\pi}{7},2\pi\right\}$. Only angles in Quadrants I or III could possibly yield a point in Quadrant I. So we need only consider the values $\left\{\frac{2\pi}{7},\frac{8\pi}{7},\frac{10\pi}{7}\right\}$.

$r=\left(\frac{5\pi}{4}-\frac{2\pi}{7}\right)\sin\frac{2\pi}{7}=\frac{27\pi}{28}\sin\frac{2\pi}{7}$, which is (+ times +) positive. A positive value of *r* with θ in Quadrant I yields a point in Quadrant I.

$r=\left(\frac{5\pi}{4}-\frac{8\pi}{7}\right)\sin\frac{8\pi}{7}=\frac{3\pi}{28}\sin\frac{8\pi}{7}$, which is (+ times –) negative. A negative value of *r* with θ in Quadrant III yields a point in Quadrant I.

$r=\left(\frac{5\pi}{4}-\frac{10\pi}{7}\right)\sin\frac{10\pi}{7}=-\frac{5\pi}{28}\sin\frac{10\pi}{7}$, which is (– times –) positive. A positive value of *r* with θ in Quadrant III yields a point in Quadrant III.

17. [C] $k=e^{\frac{5\pi}{3}i}=\text{cis}\left(\frac{5\pi}{3}\right)=\text{cis}\left(300°\right)$. Using DeMoivre's theorem, $k^{\frac{1}{4}}=\left(\text{cis}\left(300°\right)\right)^{\frac{1}{4}}=\text{cis}\left(75°\right)$.

(Although there are 3 other fourth roots, this is the principal root, which is what the notation $\sqrt{\ }$ implies.)

18. [B] Let $x=100A+10B+C$, where *A*, *B*, and *C* are digits. Then $y=100C+10B+A$ and $z=100B+10A+C$. Looking at the second and third facts,

$$\begin{aligned}100A+10B+C-100B-10A-C&=450\\90A-90B&=450\\A-B&=5\end{aligned}$$

$$\begin{aligned}100C+10B+A+100B+10A+C&=1085\\11A+110B+101C&=1085\end{aligned}$$

Looking at the first fact, $$\begin{aligned}200&<100C+10B+A-100A-10B-C<300\\200&<99C-99A<300\\\frac{200}{99}&<C-A<\frac{300}{99}\end{aligned}$$. Now, because $(C-A)$ is an integer, and $\frac{200}{99}$ is slightly more than 2, and $\frac{300}{99}$ is slightly more than 3, $C-A=3$.

Manipulating, $C-A=3\ \rightarrow\ C=A+3$; $A-B=5\ \rightarrow\ B=A-5$.

Substituting, $$\begin{aligned}11A+110(A-5)+101\left(A+3\right)&=1085\\222A-247&=1085\\222A&=1332\\A&=6\end{aligned}$$, and then $C=9$ and $B=1$.

19. [D] Looking at the signs of *A* and *B*, $f(x)=|A|x^5-|B|x^4-3x^3-x^2-9$. There is one sign change initially, so $p=1$ by DesCartes' Rule of Signs. $f(-x)=-|A|x^5-|B|x^4+3x^3-x^2-9$. There are two sign changes, so *n* is either 2 or 0. Any remaining roots are nonreal, since 0 is not a root. So the possible values for *p*, *c*, and *z* are $\{p=1,\ n=2,\ z=2\}$ and $\{p=1,\ n=0,\ z=4\}$

20. [B] Sketch the graphs and inspect them, or use brute-force algebra. Solving: $\frac{x+1}{2}=\left|1-|x-2|\right|$.

$\frac{x+1}{2}=1-|x-2|$ or $\frac{x+1}{2}=-1+|x-2|$. These each yield two more equations. Solving each:

$$\frac{x+1}{2}=1-x-2 \qquad x+1=-2x-2 \qquad 3x=-3 \qquad x=-1$$

$$\frac{x+1}{2}=1+x+2 \qquad x+1=2x+6 \qquad x=7$$

$$\frac{x+1}{2}=-1+x-2 \qquad x+1=2x-6 \qquad x=-7$$

$$\frac{x+1}{2}=-1-x+2 \qquad x+1=-2x+2 \qquad 3x=1 \qquad x=\frac{1}{3}$$

Plugging the answers back in to check, only 7 and $\frac{1}{3}$ work.

21. [D]

22. [D] This is the sum of the Fibonacci sequence $1,1,2,3,5,8,13,21,34,55,...$ and the sequence of prime numbers $2,3,5,7,11,13,17,19,23,29...$

23. [C] $(4x+3)(3x-5)$

24. [A] $\sin\left(\frac{13\pi}{12}\right)=\sin\left(\frac{3\pi}{4}+\frac{\pi}{3}\right)=\left(\sin\frac{3\pi}{4}\right)\left(\cos\frac{\pi}{3}\right)+\left(\cos\frac{3\pi}{4}\right)\left(\sin\frac{\pi}{3}\right)=\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{-\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)=$ $\frac{\sqrt{2}-\sqrt{6}}{4}$. So $4\sin\left(\frac{13\pi}{12}\right)=\sqrt{2}-\sqrt{6}$.

25. [C] $1+1=2$

26. [A] $f^{-1}(2)$ is the value of *x* for which $f(x)=2$, so $A=-1$. If *f* is odd, $f(2)=-f(-2)=-1$. So $B=-1$. Then $|2A-B|=|-2-(-1)|=1$.

27. [A] From the initial information:

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Morning		M		M			
Afternoon						B	B
Night							

Now, because he does not use mouthwash at night:

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Morning		M		M		M	M
Afternoon						B	B
Night						F	F

He cannot floss three nights in a row, and he cannot use mouthwash at night.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Morning		M		M		M	M
Afternoon						B	B
Night	B				B	F	F

He cannot use mouthwash or brush at the same time three days in a row

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Morning	F	M		M	F	M	M
Afternoon	M				M	B	B
Night	B				B	F	F

He can't use mouthwash in the morning three days in a row, so he must use it in the afternoon on Wednesday.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Morning	F	M		M	F	M	M
Afternoon	M		M		M	B	B
Night	B				B	F	F

Since he brushes more than he flosses at night, there must be 2 more B's at night. There's only one way to place them to avoid 3 in a row.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Morning	F	M		M	F	M	M
Afternoon	M		M		M	B	B
Night	B	B		B	B	F	F

Finish the chart in the only possible way.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Morning	F	M	B	M	F	M	M
Afternoon	M	F	M	F	M	B	B
Night	B	B	F	B	B	F	F

28. [A] The center, focus, and endpoint of the minor axis form a right triangle. One leg measures *c* units. $(a-c)+6=a+c \rightarrow 2c=6 \rightarrow c=3$. The hypotenuse measures 7. So the length of the major axis is $2\sqrt{40}=\sqrt{160}$.

29. [A] The fourth term is the term in question. The term is ${}_8C_{(4-1)}\left(\frac{x}{4}\right)^5(2y)^3=\frac{8!}{5!3!}\left(\frac{x^5}{2^{10}}\right)\left(2^3y^3\right)=$

$2^3 \cdot 7\left(\frac{x^5}{2^{10}}\right)\left(2^3y^3\right)=\frac{7}{16}x^5y^3$.

30. [E]