1. C – The smallest rectangle has perimeter 52, the largest has perimeter 98. Closer side lengths yield greater areas and more rectangular gives less area. Thus, 25x1 and 24x25 are the area minimizing and maximizing dimensions, respectively. 600 – 25 = 575
2. C –
   8π
   $4\cos\Theta^4 - 8\cos\Theta^2 x = -3;\ (2\cos\Theta^2 - 1)(2\cos\Theta^2 - 3) = 0;\ Cos\Theta = \pm\frac{\sqrt{2}}{2}\ or \pm\frac{\sqrt{6}}{2}, where \pm\frac{\sqrt{6}}{2}$ are not possible; The solutions are $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$; The sum is 4π
3. D – A square's area is x^2, its perimeter is $4x$. $x^2 < 4x \Rightarrow x(x-4) < 0 \Rightarrow x \in (0,4)$
4. D – Instead of directly solving, subtract the first equation from the third.
5. A – We have $\cos x = \sqrt{1-\sin^2 x}$ or $\cos x = -\sqrt{1-\sin^2 x}$, at the endpoints of the interval where $\sin x \in [1/3, 2/3]$, we have $\cos x = \sqrt{8/9}, -\sqrt{8/9}, \sqrt{5/9}$, or $-\sqrt{5/9}$. So the maximum and minimum are the first and second of these values, respectively
6. C – Choice D has 1 solution; Choice B must have 1 or 3 solutions; Choice A has 4 solutions; Choice C has solutions ± 2.
7. A – $4\pi r^2 > 4 \Rightarrow r > 1/\sqrt{\pi} \Rightarrow V = 4/3\,\pi r^3 > 4/3\,\pi(1/\sqrt{\pi})^3 = 4/3\sqrt{\pi}$
8. C – The sum of the squares of the roots is (the square of the sum of the roots)-2(the sum of the roots two at a time), which is $(5/2)^2 - 2(3/2) = (25/4) - (12/4) = 13/4$
9. C – The first equation factors into $(\cos x - 1/2)(\cos x - 1/4) = 0$. $\cos x = 1/2$ at $-\pi/3, \pi/3$, and $5\pi/3$. $\cos x = 1/4$ at two points in $(-5\pi/3, 0)$, and at two points in $[0, 6.3]$
10. A – $y = kx^2/\sqrt{z}$, so $3 = k(4^2)/\sqrt{4} = 8k,\ k = 3/8$, so $9/2 = (3/8)x^2/\sqrt{9} = x^2/8 \Rightarrow x^2 = 36$
11. A – The other root must be conjugate to the first, and therefore $\frac{-2+i\sqrt{5}}{3}$. So the product of the roots is $4+5/3^2 = 1$, and the sum of the roots is $-4/3$. So our equation is equivalent to $x^2 + 4x/3 + 1 = 0$ or $3x^2 + 4x + 3 = 0$. Our discriminant is $4^2 - 4(3)(3)$.
12. D – On the way to Gob's, the plane flies at 2mph, and on the way back, it flies 2.4mph. Thus, the airplane's natural speed is 2.2mph, and the wind speed is .2mph.
13. A – Since $0 < |\sin x| \le 1$ on I, we know $g(x) \le 0$ on I. But we also know that $f(x) \ge 1$ for all x.
14. D – We get the equations $M + L = 1/3,\ M + B = 1/4,\ B + L = 1/5$. Add them together to get $2M + 2B + 2L = 1/3 + 1/4 + 1/5 = 47/60,\ M + B + L = 47/120$, so in 47 days, they can build 120 houses together.
15. E – $|x^2| = 5, |y^2| = 10$, so $|x| = \sqrt{5}, |y| = \sqrt{10}$. And in general, $|x| + |y| \le |x + y|$. In this case, we have $6 \le |x| + |y| \le |x + y|$. Note that, for certain values of x and y, $|x + y| = \sqrt{5} + \sqrt{10} > 5$, so 6 is the smallest integral value necessary.

16. D – $\cos^4 x + \frac{1}{2}(\sin 2x)^2 + \sin^4 x + x^2$ is $\cos^4 x + 2\sin^2 x\cos^2 x + \sin^4 x + x^2$, or $(\cos^2 x + \sin^2 x)^2 + x^2$ or $1 + x^2$, which has a minimum value of 1.

17. B – I is impossible since both summands must be rational and must therefore have a rational sum. II is possible (consider $z = \sqrt{2},\ w = 1/2,\ x = 2$). III is possible, for if it were not, then $\sqrt{2}^{\sqrt{2}}$ would be irrational, and so $(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = 2$ would be irrational (a contradiction).
18. C – The revenue is then $207x - 2x^2$, which is a parabola with vertex at $x = 207/4 = 51.75$, and $x = 52$ gives the highest available revenue
19. D – $(x + y\sqrt{z})^2 = \frac{3+4\sqrt{3}}{5-2\sqrt{3}}\frac{5+2\sqrt{3}}{5+2\sqrt{3}} = \frac{39+26\sqrt{3}}{13} = 3 + 2\sqrt{3}$; thus we get $(3)(2) + 2 - 3 = 5$
20. C - $(3i-3)^{2008} = [(3\sqrt{2})(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2})]^{2008} = (3\sqrt{2})^{2008}(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4})^{2008} = 18^{1004}$
21. C – The first two equations give $x = \pm 2,\ y = \pm\sqrt{5}$, and the third inequality tells us $x = 2,\ y = \sqrt{5}$
22. C – $u = h/2,\ u + t + h = 14,\ t = h + u - 4$; solving gives us $u = 3,\ h = 6,\ t = 5$
23. D – A is possible with $(a,b) = (-1,1)$; B is possible with $(a,b) = (-1/2,1)$; C is possible with $(a,b) = (0,0)$. D is impossible since $a \le a^2 \Rightarrow a \ge 1 \text{ or } a \le 0$ and $b \ge b^2 \Rightarrow 0 \le b \le 1$ and $a^2 \le b^2 \Rightarrow |a| \le |b|$. Combining these three equalities gives $-1 \le a \le 0$.
24. D – The circle must have radius less than $\sqrt{(4)(5)}$, and greater than $\sqrt{(2)(5)}$. So it's area is less than 20π and greater than 10π
25. B – Since the nanny's wage is 100$/hr, Buster must be consuming 25 grams of sugar per hour. Since he also consumes 5 gallons of juice per hour, each gallon of juice must contain 5 grams of sugar.
26. C – The inside can be replaced: $\sqrt{k + 3/2} = 3/2$, so $k = 3/4$
27. B – Here, we use the same procedure as in problem 26: $\sqrt{3/2 + k} = k$, so $k^2 - k - 3/2 = 0$. Thus $k = \frac{1 \pm \sqrt{7}}{2}$, but the negative solution is erroneous.
28. B – It would take 15 days for 10 Seths to eat 15 gallons of ice cream, and it would take 14 Seths (10/14) that amount of time. Thus, it would take (150/14) days, or roughly 11 days.
29. A – $x^2 + y^2 + z^2 + 2xy + 2xz + 2yz = 225$. So $121 + (Surface\ Area) = 225$
30. E – Milton will run $4k/60$ miles in the first k minutes; Astro will run $\frac{1+2+3+...+k}{60}$ miles in the first k minutes. Thus, $4k = k(k+1)/2$, so $k^2 = 7k$, so they will run for 7 minutes, and $28/60$ miles