

For each question, the answer, "E. NOTA" is defined as "None of the above answers is correct."

- A rectangle with integral side lengths has a perimeter that is less than 100 and greater than 50. What is the positive difference between its maximum possible area and its minimum possible area?
A. 431 B. 469 C. 575 D. 601 E. NOTA
- Find the sum of the solutions when the domain is $0 \leq \Theta \leq 2\pi$: $4 \cos \Theta^4 - 8 \cos^2 \Theta = -3$
A. 2π B. $\frac{11\pi}{2}$ C. 4π D. $\frac{15\pi}{12}$ E. NOTA
- A square's area is less than its perimeter if and only if it has side length x , where
A. $1 < x < 2$ B. $0 < x < 2$ C. $1 < x < 4$ D. $0 < x < 4$ E. NOTA
- $a + b - c = 3$, $a + 2b - 3c = 5$, and $2a + 3b - 3c = 8$. What is $a + 2b - 2c$?
A. 2 B. 3 C. 4 D. 5 E. NOTA
- If $\frac{1}{3} \leq \sin x \leq \frac{2}{3}$, and $x \in$ the set of real numbers, what is the positive difference between the minimum value and the maximum value of $\cos x$?
A. $\frac{4\sqrt{2}}{3}$ B. $\frac{2\sqrt{5}}{3}$ C. $\frac{1}{3}$ D. $\frac{2\sqrt{2} - \sqrt{5}}{3}$ E. NOTA
- Which equation has exactly two distinct real solutions?
A. $x^4 - 5x^2 + 6 = 0$ B. $2x^3 - 4x^2 + 11x - 13 = 7$ C. $x^8 - 256 = 0$
D. $x^2 - 3x + 2.25 = 0$ E. NOTA
- A sphere with surface area greater than 4 has volume V , such that
A. $V > \frac{4\sqrt{\pi}}{3\pi}$ B. $V > 8$ C. $V > \frac{8\sqrt{\pi}}{\pi}$ D. $V > 16$ E. NOTA
- The equation $2z^4 - 5z^3 + 3z^2 + 4z = 1$ has four roots. What is the sum of the squares of these roots?
A. $\frac{37}{4}$ B. 8 C. $\frac{13}{4}$ D. 28 E. NOTA

9. How many values of x satisfy $-\cos^2 x + \frac{3}{4}\cos x = \frac{1}{8}$ and $\frac{-5\pi}{3} < x \leq 6.3$
 A. 4 B. 6 C. 7 D. 8 E. NOTA
10. If y is directly proportional to the square of x and inversely proportional to the square root of z .
 $y = 3$ when $x = z = 4$, what is $|x|$ when $z = 2y = 9$?
 A. 6 B. 9 C. 12 D. 36 E. NOTA
11. A quadratic polynomial with real integer coefficients has one root equal to $\frac{-2 - i\sqrt{5}}{3}$. What is the value of the discriminant of this quadratic? Assume the polynomial is in its most reduced form.
 A. -20 B. $-2\sqrt{5}$ C. $-2i\sqrt{5}$ D. $2\sqrt{5}$ E. NOTA
12. An airplane flying from Tobias' house to Gob's house can complete the journey in six hours. The flight from Gob's house to Tobias' house takes five hours. If the distance between the two houses is 12 miles, at what rate is the wind blowing towards Tobias' house?
 A. 12 mph B. 0.5 mph C. 0.4 mph D. 0.2 mph E. NOTA
13. Consider the interval $I = (0, \pi) \cup (\pi, 2\pi) \cup (2\pi, 3\pi) \cup \dots$. If $f(x) = |\sin(\ln x)| + 1$ and $g(x) = \ln|\sin x|$, then $f(x) > g(x)$
 A. For all $x \in I$ B. For all but 2 values of $x \in I$ C. II, I, III
 D. For finitely many $x \in I$ E. NOTA
14. Michael and Lindsay can build a house in 3 days, Michael and Buster can build a house in 4 days, and Lindsay and Buster can build a house in 5 days. How many days would it take the three of them working together to build 47 houses?
 A. 30 B. 60 C. 90 D. 120 E. NOTA
15. Given that $x^2 = 3 + 4i$ and $y^2 = 6 + 8i$, what is the smallest integer which must be greater than $|x + y|$?
 A. 1 B. 3 C. 5 D. 7 E. NOTA
16. What is the minimum value of $\cos^4 x + \frac{1}{2}(\sin 2x)^2 + \sin^4 x + x^2$?
 A. 0 B. $\frac{1}{2}$ C. $\frac{\sqrt{2}}{2}$ D. 1 E. NOTA

17. w and x are rational; y and z are irrational. Which of the following is/are possible?

I. $\frac{w}{x} + \frac{x}{w} = z$

II. $|z|^{\frac{1}{w}} = x$

III. $y^z = x$

A. I, II, III

B. II, III

C. II

D. None

E. NOTA

18. If x units of an item are sold, then the price of this item will be $(207 - 2x)$ dollars for each unit.

How many units should be sold to maximize revenue? (Assume only whole units.)

A. 104

B. 103

C. 52

D. 51

E. NOTA

19. Written in its most reduced form, $\frac{\sqrt{3+4\sqrt{3}}}{\sqrt{5-2\sqrt{3}}} = \sqrt{x+y\sqrt{z}}$. Evaluate $xy + y - z$, where z is not divisible by the square of any prime.

A. 2

B. 3

C. 4

D. 5

E. NOTA

20. Evaluate $(3i - 3)^{2008}$.

A. 3^{2008}

B. $12^{1004}i$

C. 18^{1004}

D. $6^{2008}(i+1)$

E. NOTA

21. $x^2 + y^2 = 9$, $2x^2 - y^2 = 3$, $x + y > 2$. Solve for xy .

A. 6

B. -6

C. $2\sqrt{5}$

D. $-2\sqrt{5}$

E. NOTA

22. A three-digit number has its units digit equal to half the hundreds digit. The sum of all three digits is 14. The tens digit is 4 less than the sum of the hundreds and units digits. What is the product of the digits of the number?

A. 28

B. 60

C. 90

D. 192

E. NOTA

23. Given that $a \leq a^2 \leq b^2 \leq b$, an impossible value for a would be

A. -1

B. $-\frac{1}{2}$

C. 0

D. 2

E. NOTA

24. A circle and an ellipse share a center. The ellipse is given by $\frac{(x-1)^2}{2} + \frac{(y-3)^2}{4} = 5$. The circle intersects four times with the ellipse, and has area less than a and greater than b . Then $a - b$ is always less than

A. π

B. 3π

C. 6π

D. 10π

E. NOTA

25. Lucille is buying juice for Buster. For every gram of sugar Buster consumes, Lucille must spend 4 dollars to pay for a nanny to watch over Buster and make sure he doesn't get out of control. If Buster drinks 5 gallons of juice per hour, and the nanny's wage is 100 dollars per hour, how many grams of sugar are there in each gallon of juice? (Buster only consumes sugar in juice... he loves juice).
- A. 4 B. 5 C. 10 D. 20 E. NOTA
26. Solve for k : $\sqrt{k + \sqrt{k + \sqrt{k + \sqrt{k + \dots}}} = \frac{3}{2}$
- A. The empty set B. $\frac{3}{2}$ C. $\frac{3}{4}$ D. $\frac{3}{16}$ E. NOTA
27. Solve for k : $\sqrt{\frac{3}{2} + \sqrt{\frac{3}{2} + \sqrt{\frac{3}{2} + \sqrt{\frac{3}{2} + \dots}}} = k$
- A. $\frac{1 \pm \sqrt{7}}{2}$ B. $\frac{1 + \sqrt{7}}{2}$ C. $\frac{\sqrt{7}}{2}$ D. $\frac{3}{2}$ E. NOTA
28. 10 Seths can eat ten gallons of ice cream in ten days, how many days (to the nearest day) would it take 14 Seths to eat 15 gallons of ice cream?
- A. 10 B. 11 C. 22 D. 24 E. NOTA
29. A right rectangular prism has dimensions x , y , and z , where $x + y + z = 15$. If the prism's main diagonal has length 11, what is the total surface area of the prism?
- A. 104 B. 125 C. 165 D. 214 E. NOTA
30. Milton and Astro are very good dogs, and they are racing, and Astro's speed is 1 mph in the first minute, 2 mph in the second minute, 3 mph in the third minute, etc. Milton runs at a constant 4 mph. How many miles will Astro run before he catches up with Milton, assuming they start running at the same time and place?
- A. $\frac{3}{5}$ B. $\frac{3}{10}$ C. $\frac{1}{5}$ D. $\frac{1}{10}$ E. NOTA