Mu Alpha Theta 2008 Nationals  

Alpha Functions

The abbreviation "NOTA" means "None of These Answers."

1. For \( f(x) = x^2 + x \) then \( f(x-1) = \)

A. \( x^2 + x \)  
B. \( x^2 - x \)  
C. \( x^2 + x - 1 \)  
D. \( x^2 - x + 1 \)  
E. NOTA

2. For which function is it NOT true that \( f(-x) = f(x) \) for all real values of \( x \) ?

A. \( f(x) = \sin^2 x \)  
B. \( f(x) = \cos x \)  
C. \( f(x) = e^{2x} \)  
D. \( f(x) = ex^2 - 1 \)  
E. NOTA

3. For \( f(x) = \sec^2 x \) find \( f\left( \frac{\pi}{4} \right) \).

A. 1  
B. \( \sqrt{2} \)  
C. 2  
D. 4  
E. NOTA

4. The function \( d(x,y) \) is defined as the positive distance between the point \( (x,y) \) and the line with equation \( y = \frac{4}{3}x - 3 \), on the coordinate plane. Give the value of \( d(0,5) \).

A. 8  
B. \( \frac{24}{5} \)  
C. \( \frac{6}{5} \)  
D. \( \frac{1}{5} \)  
E. NOTA

5. For \( f(x) = 4^{x+1} \) and \( g(x) = 2^{x-1} \) let \( 2^k = f(10)\cdot g(12) \). Which is the value of \( k \)?

A. 34  
B. 33  
C. 31  
D. 22  
E. NOTA

6. Let \( g(x) = \sqrt{1 - \left( x - \frac{\pi}{2} \right)^2} \). For \( 0 < x < \pi \) which function shares a common maximum point with \( g \)?

A. \( y = \sin x \)  
B. \( y = \frac{\pi}{2} \sin x \)  
C. \( y = \frac{\pi}{2} \cos x \)  
D. \( y = \sqrt{\frac{\pi}{2} - x} \)  
E. NOTA
7. If \( f(g(x)) = g(f(x)) = x \) for all real values of \( x \) then for \( f(x) = \sqrt[3]{x+1} \) find \( g(2) \).

A. \( \sqrt[3]{3} \)  B. \( \sqrt[3]{7} \)  C. 1  D. 7  E. NOTA

8. \( f(x) = \begin{cases} \frac{1}{x} & \text{for } |x| < \frac{1}{2} \\ \frac{1}{2} + \frac{1}{x} & \text{for } |x| \geq \frac{1}{2} \end{cases} \)

Give the value of \( f\left(f\left(-\frac{1}{3}\right)\right) \).

A. \(-3\)  B. \(-\frac{5}{2}\)  C. \(\frac{1}{10}\)  D. \(\frac{1}{6}\)  E. NOTA

9. For \( f(x) = 9 \) find the value of \( f(f(x-1)) \).

A. 8  B. 9  C. 17  D. 18  E. NOTA

10. If \( f(x-2) = \sqrt{x} + x \) then give the value of \( f(14) - f(7) \).

A. \(\sqrt{7} - 7\)  B. 8  C. \(\sqrt{14} - \sqrt{7} + 7\)  D. 26  E. NOTA

11. Let \( f(x) = \sin x \), \( g(x) = \cos x \). For which of the following values of \( x \) is

\[ f(x) + g(x) = \frac{f\left(\frac{\pi}{4}\right)}{g\left(\frac{\pi}{4}\right)} \]?

A. \(\frac{\pi}{4}\)  B. \(\frac{\pi}{3}\)  C. \(\frac{\pi}{2}\)  D. \(\frac{2\pi}{3}\)  E. NOTA

12. Let \( f(n) \) be the coefficient of the \( n \)th term of the expansion of \( (x-1)^{10} \). The domain of \( f \) is the set of integers \( n \) such that \( 0 < n < 12 \). Give the value of \( f(3) \div f(9) \).

A. \(-12\)  B. \(-1\)  C. 1  D. 12  E. NOTA
13. For \( f(x) = \sqrt{x + 1 + 2\sqrt{x}} \), find the value of \( f(7) = \sqrt{R + S} \). Give the value of \( R\cdot S \).
   A. 5       B. 6       C. 7       D. 8       E. NOTA

14. For \( x > 0 \) and \( f(x) = \log(x) \) if \( f(k) + f(k + 2) = f(48) - f(2) \) then \( \log_2\left(\frac{1}{k}\right) = \)
   A. -4       B. -2       C. 2       D. 4       E. NOTA

15. The angles of a convex heptagon are in an arithmetic progression. For the smallest angle, \( x^\circ \), \( f(x) \) gives the degree measure of the largest angle. The domain of \( f \) is positive real numbers. If the value of \( f(101) \) is rounded to the nearest degree, then give the sum of the digits of the result.
   A. 12       B. 11       C. 10       D. 8       E. NOTA

16. The function \( f(x) = x^2 - 3x + c \) has roots \( r_1 \) and \( r_2 \). The function \( g(x) = x^3 - 10x^2 + 7x - 1 \) \((k \text{ and } c \text{ are real constants})\) has roots \( r_1, r_2, \) and \( r_3 \). Give the value of \( r_3 \).
   A. -21       B. -7       C. 7       D. 21       E. NOTA

17. Let \( f(x) \) be the number (amount) of positive integral factors of \( x \), for \( x > 0 \).
   Give the least odd value of \( x \) for which \( f(x) = 5 \).
   A. 243       B. 125       C. 81       D. 27       E. NOTA

18. A fair coin is flipped \( n \) times. The result of each coin flip is either heads or tails. The function \( P(n) \) gives the probability that there is at least one "head" showing. Give the least value of \( n \) so that \( P(n) > \frac{15}{16} \).
   A. 4       B. 5       C. 6       D. 8       E. NOTA
19. For \(2n\) people sitting in a circle (with seats numbered 1 through \(2n\)) the function \(f(n)\) gives the seat number of the person opposite the seat numbered \(n\). What is the value of \(f(10)\)?

A. 20  
B. 19  
C. 2  
D. 1  
E. NOTA

20. For \(f(x) = \ln x\) and \(g(x) = e^x\), which value is equal to 1?

A. \(f(1)g(1)\)  
B. \(\frac{f(1)}{g(1)}\)  
C. \(f^{-1}(g(1))\)  
D. \(f(g(1))\)  
E. NOTA

21. For \(f(x) = 8\sqrt{2} - 8i\sqrt{2}\) which is not a fourth root of \(f(x)\)? (\(cis \theta = \cos \theta + i \sin \theta\))

A. \(2\text{cis}\frac{31\pi}{16}\)  
B. \(2\text{cis}\frac{15\pi}{16}\)  
C. \(2\text{cis}\frac{23\pi}{16}\)  
D. \(2\text{cis}\frac{5\pi}{16}\)  
E. NOTA

22. What is the domain over reals of \(f(g(x))\) for \(f(x) = \frac{\sqrt{x-1}}{x^2-4}\) and \(g(x) = x^2 - 1\)?

A. \((-\infty, -\sqrt{3}) \cup (-\sqrt{3}, -\sqrt{2}] \cup [\sqrt{2}, \sqrt{3}) \cup (\sqrt{3}, \infty)\)  
B. \((-\sqrt{3}, -\sqrt{2}) \cup (\sqrt{2}, \sqrt{3})\)  
C. \([1, 2) \cup (2, \infty)\)  
D. \((1, 2) \cup (2, \infty)\)  
E. NOTA

23. The function \(f(t) = \frac{200}{1 + e^{-0.1t}}\) gives the number of people who have heard a rumor at time \(t \geq 0\) hours, where \(t = 0\) is midnight on January 1, 2008. At what time have \(\frac{200\sqrt{e}}{1 + \sqrt{e}}\) people (approximately 124.5 people – one person is in the process of being told) heard the rumor? (All times are on January 1, 2008.)

A. 5:00 AM  
B. 6:15 AM  
C. 7:30 AM  
D. 8:20 AM  
E. NOTA

24. For \(f(x) = \log_x(x^2 + 2x + 1)\) and \(g(x) = 3 \cdot \log_x(x - 3)\), give the value of \(f(4)g(5)\).

A. 2  
B. 3  
C. 4  
D. 5  
E. NOTA
25. For \( n > 1 \), in a survey of \( n \) people, \( f(n) \) gives the number of people who like coffee, and \( f(n) + 6 \) gives the number of people who like tea. If everyone surveyed liked coffee and/or tea, and \( f(20) = 12 \) then how many of those surveyed like both coffee and tea?

A. 24  
B. 16  
C. 10  
D. 6  
E. NOTA

26. A ball is dropped from a height of \( x \) feet \( (x \geq 0) \) and the distance it travels totally is \( f(x) \) feet. The ball rebounds \( \frac{4}{5} \) of its previous height after each bounce. If \( f(k) = 7200 \) feet then give the value of \( k \) in feet.

A. 1440  
B. 860  
C. 800  
D. 720  
E. NOTA

27. For the restricted range \( \left( -\frac{\pi}{2}, \frac{\pi}{2} \right) \) if \( f(x) = \text{Arc tan } x \), then find \( \lim_{x \to \infty} f(x) \).

A. \( -\frac{\pi}{2} \)  
B. 0  
C. 1  
D. \( \frac{\pi}{2} \)  
E. NOTA

28. Two circles have their centers on the \( x \)-axis, and radii 6. The centers are moving apart at a constant rate. Let \( D(t) \) be the distance between the centers at time \( t \) for \( t \geq 0 \). If \( D(0) = 0 \) and \( D(1) = 0.5 \) then which statement below indicates the condition of the circles being tangent to each other?

A. \( D(6) = 3 \)  
B. \( D(12) = 6 \)  
C. \( D(6) = 0 \)  
D. \( D(24) = 12 \)  
E. NOTA

29. \( f(x) = \sqrt{x^2 - 2x + 1} \). If \( a < b < 0 \) then which is equal to \( f(a) - f(b) \)?

A. \( (a + b) + 1 \)  
B. \( b - a \)  
C. \( a + b \)  
D. \( -a - b \)  
E. NOTA

30. Let set \( S \) be \{2, 4, 6, 8, 16\} let \( f(n) \) be the product of \( n \) randomly chosen distinct members of set \( S \). Give the probability that \( f(3) < f(2) \).

A. \( \frac{1}{50} \)  
B. \( \frac{1}{20} \)  
C. \( \frac{3}{20} \)  
D. \( \frac{3}{50} \)  
E. NOTA