The abbreviation "NOTA" means "None of These Answers."

1. For $f(x)=x^2+x$ then $f(x-1)=$

A. x^2+x B. x^2-x C. x^2+x-1 D. x^2-x+1 E. NOTA

2. For which function is it NOT true that $f(-x)=f(x)$ for all real values of x ?

A. $f(x)=\sin^2 x$ B. $f(x)=\cos x$ C. $f(x)=e^{2x}$

D. $f(x)=ex^2-1$ E. NOTA

3. For $f(x)=\sec^2 x$ find $f\left(\frac{\pi}{4}\right)$.

A. 1 B. $\sqrt{2}$ C. 2 D. 4 E. NOTA

4. The function $d(x,y)$ is defined as the positive distance between the point (x,y) and the line with equation $y=\frac{4}{3}x-3$, on the coordinate plane . Give the value of $d(0,5)$.

A. 8 B. $\frac{24}{5}$ C. $\frac{6}{5}$ D. $\frac{1}{5}$ E. NOTA

5. For $f(x)=4^{x+1}$ and $g(x)=2^{x-1}$ let $2^k=f(10)\bullet g(12)$. Which is the value of k ?

A. 34 B. 33 C. 31 D. 22 E. NOTA

6. Let $g(x)=\sqrt{1-\left(x-\frac{\pi}{2}\right)^2}$. For $0<x<\pi$ which function shares a common maximum point with g ?

A. $y=\sin x$ B. $y=\frac{\pi}{2}\sin x$ C. $y=\frac{\pi}{2}\cos x$

D. $y=\sqrt{\frac{\pi}{2}-x}$ E. NOTA

7. If $f(g(x)) = g(f(x)) = x$ for all real values of x then for $f(x) = \sqrt[3]{x+1}$ find $g(2)$.

A. $\sqrt[3]{3}$ B. $\sqrt[3]{7}$ C. 1 D. 7 E. NOTA

8. $f(x) = \begin{cases} \dfrac{1}{x} & for\ |x| < \dfrac{1}{2} \\ \dfrac{1}{2} + \dfrac{1}{x} & for\ |x| \geq \dfrac{1}{2} \end{cases}$ Give the value of $f\left(f\left(-\dfrac{1}{3}\right)\right)$.

A. -3 B. $-\dfrac{5}{2}$ C. $\dfrac{1}{10}$ D. $\dfrac{1}{6}$ E. NOTA

9. For $f(x) = 9$ find the value of $f(f(x-1))$.

A. 8 B. 9 C. 17 D. 18 E. NOTA

10. If $f(x-2) = \sqrt{x} + x$ then give the value of $f(14) - f(7)$.

A. $\sqrt{7} - 7$ B. 8 C. $\sqrt{14} - \sqrt{7} + 7$ D. 26 E. NOTA

11. Let $f(x) = \sin x$, $g(x) = \cos x$. For which of the following values of x is $f(x) + g(x) = \dfrac{f\left(\dfrac{\pi}{4}\right)}{g\left(\dfrac{\pi}{4}\right)}$?

A. $\dfrac{\pi}{4}$ B. $\dfrac{\pi}{3}$ C. $\dfrac{\pi}{2}$ D. $\dfrac{2\pi}{3}$ E. NOTA

12. Let $f(n)$ be the coefficient of the n^{th} term of the expansion of $(x-1)^{10}$. The domain of f is the set of integers n such that $0 < n < 12$. Give the value of $f(3) \div f(9)$.

A. -12 B. -1 C. 1 D. 12 E. NOTA

13. For $f(x)=\sqrt{x+1+2\sqrt{x}}$, find the value of $f(7)=\sqrt{R}+\sqrt{S}$.
Give the value of $R \bullet S$.

A. 5 B. 6 C. 7 D. 8 E. NOTA

14. For $x>0$ and $f(x)=\log(x)$ if $f(k)+f(k+2)=f(48)-f(2)$ then $\log_2\left(\frac{1}{k}\right)=$

A. -4 B. -2 C. 2 D. 4 E. NOTA

15. The angles of a convex heptagon are in an arithmetic progression. For the smallest angle, x°, $f(x)$ gives the degree measure of the largest angle. The domain of f is positive real numbers. If the value of $f(101)$ is rounded to the nearest degree, then give the sum of the digits of the result.

A. 12 B. 11 C. 10 D. 8 E. NOTA

16. The function $f(x)=x^2-3x+c$ has roots r_1 and r_2. The function $g(x)=x^3-10x^2+kx-7c$ (k and c are real constants) has roots r_1, r_2, and r_3.
Give the value of r_3.

A. -21 B. -7 C. 7 D. 21 E. NOTA

17. Let $f(x)$ be the number (amount) of positive integral factors of x, for $x>0$.
Give the least **odd** value of x for which $f(x)=5$.

A. 243 B. 125 C. 81 D. 27 E. NOTA

18. A fair coin is flipped *n* times. The result of each coin flip is either heads or tails. The function $P(n)$ gives the probability that there is at least one "head" showing. Give the least value of n so that $P(n)>\frac{15}{16}$.

A. 4 B. 5 C. 6 D. 8 E. NOTA

19. For $2n$ people sitting in a circle (with seats numbered 1 through 2n) the function $f(n)$ gives the seat number of the person opposite the seat numbered n. What is the value of $f(10)$?

A. 20 B. 19 C. 2 D. 1 E. NOTA

20. For $f(x)=\ln x$ and $g(x)=e^x$, which value is equal to 1 ?

A. $f(1)\bullet g(1)$ B. $\dfrac{f(1)}{g(1)}$ C. $f^{-1}(g(1))$ D. $f(g(1))$ E. NOTA

21. For $f(x)=8\sqrt{2}-8i\sqrt{2}$ which is not a fourth root of $f(x)$? $(cis\theta=\cos\theta+i\sin\theta)$

A. $2cis\dfrac{31\pi}{16}$ B. $2cis\dfrac{15\pi}{16}$ C. $2cis\dfrac{23\pi}{16}$ D. $2cis\dfrac{5\pi}{16}$ E. NOTA

22. What is the domain over reals of $f(g(x))$ for $f(x)=\dfrac{\sqrt{x-1}}{x^2-4}$ and $g(x)=x^2-1$?

A. $(-\infty,-\sqrt{3})\cup(-\sqrt{3},-\sqrt{2}]\cup[\sqrt{2},\sqrt{3})\cup(\sqrt{3},\infty)$
B. $(-\sqrt{3},-\sqrt{2})\cup(\sqrt{2},\sqrt{3})$
C. $[1,2)\cup(2,\infty)$
D. $(1,2)\cup(2,\infty)$
E. NOTA

23. The function $f(t)=\dfrac{200}{1+e^{-0.1t}}$ gives the number of people who have heard a rumor at time $t\geq 0$ hours, where $t=0$ is midnight on January 1, 2008. At what time have $\dfrac{200\sqrt{e}}{1+\sqrt{e}}$ people (approximately 124.5 people – one person is in the process of being told) heard the rumor? (All times are on January 1, 2008.)

A. 5:00 AM B. 6:15 AM C. 7:30 AM D. 8:20 AM E. NOTA

24. For $f(x)=\log_x(x^2+2x+1)$ and $g(x)=3\cdot\log_x(x-3)$, give the value of $f(4)\bullet g(5)$.

A. 2 B. 3 C. 4 D. 5 E. NOTA

25. For $n>1$, in a survey of n people, $f(n)$ gives the number of people who like coffee, and $f(n)+6$ gives the number of people who like tea. If everyone surveyed liked coffee and/or tea, and $f(20)=12$ then how many of those surveyed like both coffee and tea?

A. 24 B. 16 C. 10 D. 6 E. NOTA

26. A ball is dropped from a height of x feet ($x\geq 0$ and the distance it travels totally is $f(x)$ feet. The ball rebounds $\frac{4}{5}$ of its previous height after each bounce. If $f(k)=7200$ feet then give the value of k in feet.

A. 1440 B. 860 C. 800 D. 720 E. NOTA

27. For the restricted range $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ if $f(x)=Arc\tan x$, then find $\lim_{x\to\infty} f(x)$.

A. $-\frac{\pi}{2}$ B. 0 C. 1 D. $\frac{\pi}{2}$ E. NOTA

28. Two circles have their centers on the *x*-axis, and radii 6. The centers are moving apart at a constant rate. Let $D(t)$ be the distance between the centers at time t for $t\geq 0$. If $D(0)=0$ and $D(1)=0.5$ then which statement below indicates the condition of the circles being tangent to each other?

A. $D(6)=3$ B. $D(12)=6$ C. $D(6)=0$ D. $D(24)=12$ E. NOTA

29. $f(x)=\sqrt{x^2-2x+1}$. If $a<b<0$ then which is equal to $f(a)-f(b)$?

A. $(a+b)+1$ B. $b-a$ C. $a+b$ D. $-a-b$ E. NOTA

30. Let set S be $\{2, 4, 6, 8, 16\}$ let $f(n)$ be the product of n randomly chosen distinct members of set S. Give the probability that $f(3)<f(2)$.

A. $\frac{1}{50}$ B. $\frac{1}{20}$ C. $\frac{3}{20}$ D. $\frac{3}{50}$ E. NOTA