Sequences and Series

Directions: Select the best answer for each multiple choice questions. Recall that there is an E answer meaning "none of these."

1) If
$$a_{n+1} = n^2 - 2n + 2$$
, then $a_{10} =$
(A) 122 (B) 82 (C) 65 (D) 101
2) If the first term of an arithmetic sequence is 5, the last term is 90, and there are 38 terms, find the common difference. (rounded to the nearest tenth)
(A) 2.2 (B) 2.3 (C) 2.4 (D) 2.5
3) The sum of the series: $\sum_{n=1}^{4} \frac{3}{n(n+1)(n+2)}$, is closest to which of the following:
(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$
4) Find the number of terms in the expansion of $(x + y + xz)^7$
(A) 36 (B) 12 (C) 48 (D) 45
5) Find the number of zeros in 1000!
(A) 248 (B) 247 (C) 250 (D) 249
6) Find the combined sum of the 10th triangular number, 10th square number, 10th rectangular number, and 10th pentagonal number.
(A) 555 (B) 410 (C) 465 (D) 450

7) Find the 5^{th} term in the 25^{th} row of pascal's triangle.

(A) 10626 (B) 53130 (C) 32760 (D) 16384

8) Find the sum of the infinite sequence: $\frac{2}{3} + \frac{3}{9} + \frac{4}{27} + \frac{5}{81} + \dots$

(A) 5/3 (B) 3/2 (C) 4/3 (D) 5/2

9) Find the equivalent of $3.12\overline{7}$ as an improper fraction

(A) 563/180 (B) 23/180 (C) 617/198 (D) 23/198

10) Find $(1+i)^{20}$

(A) -1024 (B) -1024i (C) 1024 (D) 1024i

11) Find the 103rd digit, after the decimal point, in the expansion of $\frac{1}{7}$.

(A) 1 (B) 4 (C) 2 (D) 8

12) If an infinite harmonic series is also an arithmetic series, then the series MUST:

(A) Diverge (B) Converge (C) Have initial term = 0(D) Have initial term = 1

13) What is the last digit in 1333^{1333} ?

- (A) 1 (B) 9 (C) 7 (D) 3
- 14) Which of the following series converge:

A =
$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2 + 2n}}$$
 B = $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + 2n}}$ C = $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2 - 1}}$
(A) A&B (B) B&C (C) A&C (D) A, B, and C

15) The Lucas sequence is 1, 3, 4, 7, 11, 18, 29, 47, . . . Let L_n be a term of the Lucas Sequence. Describe L_n in terms of the Fibonacci Sequence for $n \ge 2$.

(A) $L_n = F_{n+1} + F_{n-1}$ (B) $L_n = F_{n+2} - F_n$ (C) $L_n = F_n + F_{n+1}$ (D) $L_n = F_{n+1} - F_n$

16) Find the sum of the next two terms in the sequence: $50, 40, 32, 26, \ldots$

(A) 38 (B) 34 (C) 42 (D) 36

17) Find x such that $x = 2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{\dots}}}}$ (A) $-1 + \sqrt{2}$ (B) $2 + \sqrt{2}$ (C) $1 + \sqrt{2}$ (D) $\sqrt{2}$

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18) The following program is executed:

N = 1 S = 0Label = 1 K = 2n-3 S = S + K N = N + 2If N < 5, Goto 1 Print S

If the above program is executed, the output will be:

- (A) 2 (B) 5 (C) 3 (D) -1 19) Evaluate: $\sum_{n=1}^{20} (n^2 - 2n + 10)$ (A) 8610 (B) 2030 (C) 2230 (D) 2650
- 20) The geometric mean between 12 and 3 is:
- (A) 36 (B) 4 (C) 3 (D) 9

21) Mr. X pizza company is featuring their "extra extra extra large pizza" The pizza cutter decides to cut the pizza into as many slices as possible. If the pizza cutter makes 300 slices, all of which are in the form of linear cuts not necessarily going through the center, what is the greatest number of slices that can be made?

(A) 45150 (B) 44850 (C) 45000 (D) 45149

22) The measures of the angles of a convex pentagon are in arithmetic progression. If the largest angle is 150°, find the positive difference of the largest and smallest angle.

(A) 70 (B) 66 (C) 78 (D) 84

23) A vacuum pump removes 30% of the air in a container with each stroke. What is the minimum number of strokes needed to reduce the amount of air in the container to less than 10%?

(A) 6 (B) 7 (C) 8 (D) 9

24) The year 2002 is the first palindrome of the new millennium. What is the sum of the units digits of the palindrome years of the previous millennium?

(A) 9 (B) 8 (C) 10 (D) 7

25) Evaluate $\sum_{k=1}^{\infty} \cos^{2k} x$ when $x \neq (2n \pm 1)\pi$ or $x \neq (2n)\pi$

(A) $\cot^2 x$ (B) $\cos^2 x$ (C) $\sin^2 x$ (D) $\sin^2 x - 1$

26) The first three terms of an arithmetic sequence are 76, 72, and 68. There are 2 values such that the sum of the first n terms is 448. Find the sum of these two values.

(A) 36 (B) 37 (C) 38 (D) 39

27) Find the sum of the following series: $\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \dots + \frac{1}{\sqrt{10}+\sqrt{9}}$

(A) $\sqrt{10} + 1$ (B) $\sqrt{10} - 1$ (C) 2 (D) 3

28) Evaluate: $\sum_{n=1}^{50} \frac{2}{(2n-1)(2n+1)}$ (round to nearest hundredth) (A) .97 (B) .98 (C) .99 (D) 1.00

29) Two simmers, at opposite ends of a 90-foot pool, start to swim the length of the pool, one at the rate of 3 feet per second, and the other at 2 feet per second. They swim back and forth for 12 minutes, allowing no loss of time at the turns. Find the number of times they pass each other.

(A) 24 (B) 21 (C) 20 (D) 19

30) Four positive integers are given. Select any three of these integers, find their arithmetic average, and add this result to the fourth integer. Thus the numbers 29, 23, 21, and 17 are obtained. One of the original integers is:

(A) 19 (B) 21 (C) 23 (D) 29