1. C. "Imaginary" was coined by Descartes in 1637.
2. C. The Argand Plane is a standard term.
3. D. The Fundamental Theorem of Algebra is standard material for math classes.
4. D. De Moivre's Formula.
5. D. $i^{195751} = i^{4(48937)+3} = i^3 = -i$
6. A. True. $|a+bi||c+di| = (\sqrt{a^2+b^2})(\sqrt{c^2+d^2}) = \sqrt{(ac)^2+(bc)^2+(ad)^2+(bd)^2}$

$$= |(ac-bd)+(bc+ad)i| = |(a+bi)(c+di)|$$

7. D. Simply by knowing Euler's Identity, students will know that π is an answer. But so is an angle coterminal with π, thus giving $\{(2n+1)\pi \mid n \in \mathbb{Z}\}$.
8. B. $\sqrt{-24} \cdot \sqrt{-6} = (i\sqrt{24})(i\sqrt{6}) = i^2\sqrt{144} = -12$
9. A. $\sqrt{(-24)(-6)} = \sqrt{-(-144)} = \sqrt{144} = 12$. Clearly, it is important to understand the slight but crucial difference between questions 8 and 9.
10. D. This is just the distance between (15, 0) and (0, 8). $\sqrt{225+64} = \sqrt{289} = 17$. Notably, 8-15-17 is a common Pythagorean Triple.
11. D. For the equation to be true, the exponents have to be congruent mod 4. This simply means that $6x+17 = 5x+2+4k$ (where *k* is an integer). Solving: $x = -15+4k \Rightarrow x = -1+4k$.
12. A. The sum can be any number from 2 to 12. Thus the only *S* such that $i^{S/2}$ is a positive integer is 8. So, 8 = 6+2, 5+3, 4+4, 3+5, and 2+6. So there are 5 ways to get the desired sum, and clearly there are $6^2 = 36$ total possible rolls.
13. C. De Moivre's Theorem: $i^{\frac{1}{2}} = (cis(\frac{\pi}{2}))^{\frac{1}{2}} = cis(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} + \frac{i\sqrt{2}}{2}$. Another root is $-\frac{\sqrt{2}}{2} - \frac{i\sqrt{2}}{2}$.
14. D. Note $-1 = e^{\pi i}$, so clearly $i = e^{\frac{\pi i}{2}}$. Thus $i^i = (e^{\frac{\pi i}{2}})^i = e^{-\frac{\pi}{2}} = \frac{1}{e^{\frac{\pi}{2}}}$.
15. D. All roots are complex, so we must find each root and separately take their absolute values. This is simple using the rational root theorem and synthetic division. Logically checking -1 as a root quickly reduces the polynomial to 4^{th} degree, then trying 2 and ½ give the remaining polynomial $2x^2+8$, which has two obvious purely imaginary roots.
Thus, we have |-1|+|2|+|½|+|4*i*|+|-4*i*|=1+2+½+4+4= 15/2
16. C. Using Descartes' rule of signs makes this simple: there is 1 sign change for x and no sign changes for –x, so there is exactly one positive real root and no negative real zeros. Thus the other 6 zeros must be non-real. Note: counting multiplicity allows for the possibility of double roots.
17. E. Taking our cue from number 11 at the top of the page, this problem is slightly trickier. Solving the congruence $2x^2+x+2 \equiv x^2+2x+1 \pmod{4}$ will give the correct set of numbers. More simply, knowing that for each single solution will fall into one of the classes $4k$, $4k+1$, $4k+2$, or $4k+3$, of solutions. Simply checking one number from each class (for example, plugging in 0, 1, 2, and 3) will tell whether each class is a set of solutions. As it turns out, both 2 and 3 are solutions, so any number of the form $4k+2$ or $4k+3$ (where *k* is an integer) is a solution.
18. C. Knowing the meaning of additive and multiplicative inverses, solving $-z = z^{-1} \Rightarrow -z^2 = 1 \Rightarrow z^2 = -1$ yields the two simple solutions *i* and –*i*.
19. D. The least *n* that fixes $i^{n!}$ is 4. For $n \geq 4$, $i^{n!} = 1$. Thus |1+4*i*| = $\sqrt{17}$.
20. C. Two non-equal complex numbers can have the same real part, imaginary part, argument, or absolute value. It is only necessary that they not have both the same real AND imaginary part.
21. C. This is simply the system of equations $3x+4y=10$ and $2x+5y=14$. The astute observer notices that subtracting these gives the desired equation: $-x+y=4$.

22. A. $(3e^{\frac{5\pi i}{6}})^3 = 27e^{\frac{5\pi i}{2}} = 27e^{\frac{\pi i}{2}+2\pi i} = 27i$

23. A. Area of triangle at (3, 5), (7, -2), and (-9, 11). Solve using your favorite method. Mine is $\frac{1}{2}\begin{vmatrix} 3 & 5 & 1 \\ 7 & -2 & 1 \\ -9 & 11 & 1 \end{vmatrix} = -30$. Area must be positive, so 30.

24. B. The distance between (i, 0) and (0, i) is the magnitude of their difference, (i, -i). The magnitude of (i, -i) is the square root of the dot product of this vector with itself. Recall that the dot product in complex space is the sum of the products of the coordinates of the first vector with the **complex conjugates** of the coordinates of the second vector. This fact is worth looking up. So for this vector the dot product is $i(-i)+(-i)i = 2$, so the distance is $\sqrt{2}$.
Alternatively, one could think of this simply as a 4-dimensional space, since each complex number is essentially an ordered pair in itself. So the distance between (0, 1, 0, 0) and (0, 0, 0, 1) is just $\sqrt{1^2+1^2} = \sqrt{2}$.

25. B. We immediately get $\ln(-1)-\ln(4) = \ln(x^2 - x)$, so then $\ln(-1) = \ln(4x^2 - 4x)$, which yields the perfect square trinomial $4x^2 - 4x + 1 = 0$ with root ½. Since the natural log function is defined to have complex domain and range (as in the directions at the beginning of the test), ½ is valid.

26. D. $z^4 + z^3 + z^2 + z = 0 \Rightarrow z(z^3 + z^2 + z + 1) = 0$. The roots are 0, -1, *i*, and –*i*.

27. C. $rcis(\theta) = r\cos(\theta) + ir\sin(\theta)$, so the complex conjugate is $r\cos(\theta) - ir\sin(\theta)$. Using trig identities, we get that this is equal to $r\cos(\theta) + ir\sin(-\theta) = r\cos(-\theta) + ir\sin(-\theta) = rcis(-\theta)$.

28. D. Given the matrix for 1, by squaring some simple matrices, we see $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.
From this we quickly get $a\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + b\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

29. B. $(-2+2i\sqrt{3})(-3\sqrt{3}-3i)$ factors to $\left[4\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\right]\left[6\left(\frac{-\sqrt{3}}{2}-\frac{i}{2}\right)\right] = \left[4cis\left(\frac{2\pi}{3}\right)\right]\left[6cis\left(\frac{7\pi}{6}\right)\right]$.
Using De Moivre's, we get $24cis\left(\frac{11\pi}{6}\right) = 24\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right) = 12\sqrt{3} - 12i$.

30. A. Since $\arg(z) = \theta$, we only need to know $\arg(289-289i) = -\frac{\pi}{4}$ and then use De Moivre's Formula and see that $175\cdot\left(-\frac{\pi}{4}\right) = -\frac{175\pi}{4}$ is coterminal with $\frac{\pi}{4}$.