1. The two are equivalent by the triple scalar product rule: **u•(v** x **w) = (u** x **v)•w =** 24. **A**
2. To find the A^{-1}, augment A with the identity matrix, and perform row reduction:

$$\begin{bmatrix} 1 & 2 & 3:1 & 0 & 0 \\ 4 & 5 & 6:0 & 1 & 0 \\ 7 & 8 & -1:0 & 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0:-53/30 & 13/15 & -1/10 \\ 0 & 1 & 0:\ 23/15 & -11/15 & 1/5 \\ 0 & 0 & 1:\ -1/10 & 1/5 & -1/10 \end{bmatrix}$$

Thus, 5 out of the 9 entries in A^{-1} are negative. Pr[choosing a negative entry] at random is thus $\frac{5}{9}$. **C**

3. The determinant of an upper triangular matrix is the product of the entries along the main diagonal. Thus, the determinant is (3)(-5)(8)(1)(2) = -240. **E**
4. Substituting the given points into the function yields the system of equations:
   a + b + c = 3 a + 2b + 4c = 2 a – b + c = -1
   Perform row reduction on the augmented matrix for this system to find the solutions:

$$\begin{bmatrix} 1 & 1 & 1:3 \\ 1 & 2 & 4:2 \\ 1 & -1 & 1:-1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0:2 \\ 0 & 1 & 0:2 \\ 0 & 0 & 1:-1 \end{bmatrix}$$

   From this reduced matrix, a = 2, b = 2, and c = -1. Thus a + b – c = 2 + 2 – (-1) = 5. **B**
5. Interchanging rows an odd number of times negates the determinant, and multiplying a row by a scalar multiplies the determinant by that scalar. Replacing a row by the sum of a multiple of another row and itself does not change the determinant. Thus, the new determinant is det(B)(-1)(3) = -243. **C**
6. Since $\mathbf{a} \bullet \mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\|\cos\theta = 1$, $\|\mathbf{a}\times\mathbf{b}\| = \|\mathbf{a}\|\|\mathbf{b}\|\sin\theta = 1$, $\frac{\|\mathbf{a}\times\mathbf{b}\|}{\mathbf{a}\bullet\mathbf{b}} = \frac{\|\mathbf{a}\|\|\mathbf{b}\|\sin\theta}{\|\mathbf{a}\|\|\mathbf{b}\|\cos\theta} = \tan\theta = 1$, θ = 45° and the sum of the digits is 9. **D**
7. The volume of the tetrahedron is $\frac{1}{6}$ (volume of the parallelepiped). The volume of a parallelepiped is found by computing the triple scalar product of vectors **a, b,** and **c**, where **a** = Q-P = <0,0,2>, **b** = R-P = <1,2,0>, and **c** = S-P= <3,-1, ½ >.Volume of tetrahedron = $\frac{\begin{vmatrix} 0 & 0 & 2 \\ 1 & 2 & 0 \\ 3 & -1 & 1/2 \end{vmatrix}}{6} = \frac{14}{6} = \frac{7}{3}$. **D**
8. $3A + B - AB = \begin{bmatrix} 3 & -3\pi^2/16 & 3\pi \\ 3\pi/2 - \pi^2/2 & 5 - 3\pi/4 & -2\pi - 10 \\ 2\pi - 6 & 6 + \pi/4 & 1 \end{bmatrix}$. $C_{13} = 3\pi$, and sin3π = 0. **A.**
9. A is a rotation by 45° and dilation by a scaling factor of 2. B is a horizontal shear. C is a projection onto the line defined by the vector <3/5, 4/5>. D is a reflection over the line angled at 45°. **A** is the only rotation dilation.
10. Rotations, shears, and reflections are all invertible – only projections are not. This can be seen geometrically: rotations, shears, and reflections can all be "undone" by another matrix (a rotation by α can be "undone" by a rotation by –α, a shear by a factor of *a* can be "undone" by shearing by –*a*, and reflections "undo" themselves when applied again). After projecting, we cannot return to our original unique point. **C**
11. Perform row reduction to arrive at the solution

$$\begin{bmatrix} 1 & 4 & 1 & 4 & : & 9 \\ 2 & 2 & -1 & 2 & : & -6 \\ 3 & -2 & 2 & 1 & : & 4 \\ 4 & 4 & 4 & 4 & : & 18 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & : & -2 \\ 0 & 1 & 0 & 0 & : & 1/2 \\ 0 & 0 & 1 & 0 & : & 5 \\ 0 & 0 & 0 & 1 & : & 1 \end{bmatrix}$$

   Reading off the solutions gives A = -2, B = ½, C = 5, and D =1. $ABC^2/5D$ = -5. **B**
12. Set up an augmented matrix and perform row reduction

$$\begin{bmatrix} 3 & 6 & : & 1 \\ 4 & 8 & : & 4 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & : & 1/3 \\ 0 & 0 & : & 2/3 \end{bmatrix}$$

   Clearly the system is inconsistent, therefore there are 0 solutions. **A.**

13. To be in reduced row echelon form, a matrix must have a 1 as the first nonzero element in a row, must have 0's in the column above and below the first 1 in a row, and the first 1 in a row must occur to the left of any first 1's in rows below it. Thus, only I, III, and V are in reduced row echelon form. **E**
14. Two vectors in the plane are **u** = <0-1, 1-2, 2-4> = <-1, -1, -2> and **v** = <-1-1, 0-2, 1-4> = <-2, -2, -3>. The cross product **u**x**v** = $\begin{vmatrix} i & j & k \\ -1 & -1 & -2 \\ -2 & -2 & -3 \end{vmatrix}$ **= <**-1, 1, 0>.

    So <-1, 1, 0> • <x-x_0, y-y_0, z-z_0> = 0 (i.e. the vector <-1, 1, 0> is orthogonal to any vector chosen in the plane). Computing the dot product yields -x + x_0 + y – y_0 = 0. Substituting one of the given points, -x + y = 1 is the equation of the plane. Thus A = -1, B = 1, C = 0, D = 1, and ABD = -1. **A**

15. Compute **PQ** = <-1, -1, -2>, **PR** = <-2, -2, -3>, and **PQ**x**PR = <**-1, 1, 0>. The area of a triangle is one half the magnitude of the cross product, so area = $\frac{1}{2}\sqrt{(-1)^2+1^2+0^2} = \frac{\sqrt{2}}{2}$. **C**

16. <1, -2, 3> • <1, 1, 4> = (1)(1) + (-2)(1) + (3)(4) = 11. **D**

17. By definition, $A^TA = I_n$ for orthogonal matrices. **D**.

18. A skew symmetric matrix has the property $A^T = -A$. This necessitates that all entries along the main diagonal must be zero. Thus, tr(A) = 0. **B**.

19. Subtract vertices in pairs to form vectors, and find the dot products of these pairs. A dot product of 0 indicates orthogonality and thus a right triangle. Vertices are abbreviated by P, Q, and R:

I.	II.	III.	IV.
PQ•PR = 28	**PQ•PR** = 1	**PQ•PR** = 11	**PQ•PR** = 22
PQ•QR = -15	**PQ•QR** = -2	**PQ•QR** = -75	**PQ•QR** = -51
PR•QR = -4	**PR•QR** = 0	**PR•QR** = 30	**PR•QR** = 3

    Thus, only II is a right triangle. **B**

20. <1, -2, 1> x <-2, 1, 2> = <-5, -4, -3>, and the unit vector in this direction is $1\big/\sqrt{(-5)^2+(-4)^2+(-3)^2}$ <-5, -4, -3> = $\frac{1}{\sqrt{2}}$ <-1, -4/5, -3/5>. **A.**

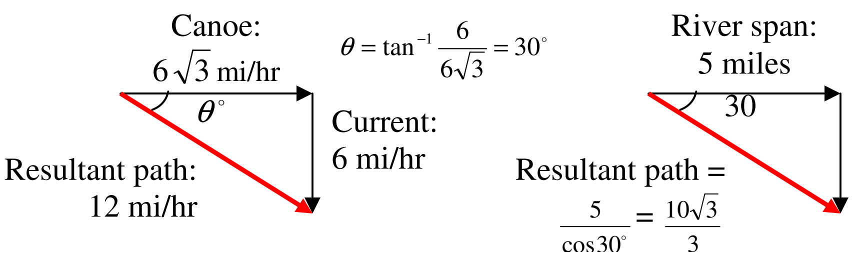
21. Using Cramer's rule to solve the system of equations A**x** = **b**, y can be found by $\frac{|A_y|}{|A|}$, where A_y is the coefficient matrix A with the y column replaced by **b**. The answer is **B.**

22. The only condition X must necessarily satisfy is det(X) = 0. Answers B and D are contradictions of this fact. X does not necessarily have to be orthogonal, so answer A is false. X also does not necessarily have to be similar to A (similar matrices satisfy the relation $S^{-1}AS = X$, and have the same rank, determinant, trace, eigenvalues, etc. So while X must have a determinant of 0, A does not necessarily need to have a determinant of 0, and thus the two are not necessarily similar). **E**

23. The eigenvalues of A are found by solving $\begin{vmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 5 \\ 0 & 0 & 6-\lambda \end{vmatrix}$=0. Since this matrix is upper triangular, the determinant is simply (1- λ)(4- λ)(6- λ). The solutions of (1- λ)(4- λ)(6- λ) = 0 are readily seen as 1, 4, and 6. The product of the smallest and largest eigenvalues is (1)(6) = 6. **C**.

24. Subtracting the two matrices on the right yields the following three quadratic equations:
    $x^2 + 5x - 6 = 0$ → (x-1)(x+6) → solutions 1, -6
    $x^2 + 6x - 7 = 0$ → (x-1)(x+7) → solutions 1, -7
    $2x^2 - 6x + 4 = 0$ → (x-1)(2x-4) → solutions 1, 2
    The only common solution is x = 1, so x =1 is the only solution to the entire system. **C**

25. The shortest time will be when Jane heads due east and allows the current to sweep her southward along the way. This time will be the shortest because all of Jane's speed is directed in an east-west direction (if she were to angle the direction of her canoe, she reduces the east-west component of her speed, and thus it takes her longer to cross the east-west 5 mile span of the river). Using vector addition, one can find the speed and distance of her resultant journey:



Thus, the distance is $\frac{10\sqrt{3}}{3}$ miles. **D**

26. Choice D is readily seen as a scalar multiple of **b**. Choices A, B, and C can be placed individually as the third row of a matrix with **a** and **b** as the first and second rows. Row reduction of these matrices shows that A and B are multiples of **a** and **b**. Choice A is 2**a** – **b**, choice B is 2**b** – **a**. Only choice C can be row reduced to a matrix without a row of 0s; row reduction with choice C results in the identity matrix. C is thus linearly independent of **a** and **b**. **C**

27. $(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} -9 & -47 & 43 \\ 22 & 46 & -55 \\ 17 & -1 & 6 \end{bmatrix}$. The sum of the entries is 22. **B**

28. $(AB)^T = B^TA^T = \begin{bmatrix} 1 & 1 & -3 \\ -3 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}$. The product of the entries is -27. **D**.

29. $\begin{vmatrix} x & 1 & 5 \\ -2 & x & 1 \\ x & -1 & x \end{vmatrix}$ $= (x^3 + x + 10) - (5x^2 - x - 2x) = x^3 - 5x^2 + 4x + 10$. The real root is x = -1. Thus, **B**.

30. L_1 is in the direction of the vector **u** = <1, 2, -1> and L_2 is in the direction of the vector **v** = <4, 1, 1>. The cosine of the angle between two vectors = $\frac{\mathbf{u} \bullet \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{4+2-1}{\sqrt{6}\sqrt{18}} = \frac{5}{\sqrt{108}} = \frac{5\sqrt{3}}{18}$. **C**.