For all of the questions on this test, NOTA should be interpreted as "None Of The Above Answers is Correct."

1. For three vectors **u**, **v**, and **w**, if **u•(v** x **w) =** 24, what does (**u** x **v)•w** equal?

A. 24 B. -24 C. $\frac{1}{24}$ D. $-\frac{1}{24}$ E. NOTA

2. Given that A=$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & -1 \end{bmatrix}$, what is the probability that an entry chosen at random in A^{-1} is less than 0?

A. $\frac{2}{3}$ B. $\frac{4}{9}$ C. $\frac{5}{9}$ D. 1 E. NOTA

3. Find $\begin{vmatrix} 3 & 0 & 0 & 0 & 0 \\ 10 & -5 & 0 & 0 & 0 \\ 14 & -2 & 8 & 0 & 0 \\ -7 & 3 & 4 & 1 & 0 \\ 3 & 1 & 0 & 1 & 2 \end{vmatrix}$.

A. 150 B. -180 C. 240 D. 0 E. NOTA

4. The graph of a function $f(x) = a + bx + cx^2$ passes through the points (1, 3), (2, 2), and (-1, -1) for some constants a, b, and c such that $abc \neq 0$. Find a+b-c.

A. 3 B. 5 C. 4 D. 0 E. NOTA

5. Given that det(B) = $\begin{vmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{vmatrix}$ = 81, what is $\begin{vmatrix} B_{31} & B_{32} & B_{33} & B_{34} \\ 3B_{21} & 3B_{22} & 3B_{23} & 3B_{24} \\ B_{11} & B_{12} & B_{13} & B_{14} \\ B_{41}+2B_{21} & B_{42}+2B_{22} & B_{43}+2B_{23} & B_{44}+2B_{24} \end{vmatrix}$?

A. 81 B. $\frac{-243}{2}$ C. -243 D. 486 E. NOTA

6. For two vectors **a** and **b**, **a** • **b** = $\|\mathbf{a}\times\mathbf{b}\|$=1**.** Given that the smaller angle between them is θ degrees, find the sum of the digits of θ.

A. 0 B. 3 C. 6 D. 9 E. NOTA

7. What is the volume of the tetrahedron with vertices P = (0, 0, 0), Q = (0, 0, 2), R = (1, 2, 0), and S = (3, -1, ½)?

A. 1 B. $\frac{11}{6}$ C. $\frac{5}{3}$ D. $\frac{7}{3}$ E. NOTA

8. $A = \begin{bmatrix} 1 & 0 & 3\pi/4 \\ \pi/2 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} \pi & 2 & 4 \\ 0 & 1 & 6 \\ 0 & \pi/4 & -1 \end{bmatrix}$. What is $\sin[C_{13}]$, where $C = 3A + B - AB$?

A. 0 B. $\frac{1}{2}$ C. $\frac{\sqrt{3}}{2}$ D. 1 E. NOTA

9. Which of the given matrices is a rotation dilation?

A. $\begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix}$ B. $\begin{bmatrix} 1 & -\frac{\sqrt{3}}{2} \\ 0 & 1 \end{bmatrix}$ C. $\begin{bmatrix} \frac{9}{25} & \frac{12}{25} \\ \frac{12}{25} & \frac{16}{25} \end{bmatrix}$ D. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ E. NOTA

10. Of the following choices, choose all that are invertible transformations.

I. Rotations II. Shears III. Projections IV. Reflections

A. I only B. I and IV only C. I, II, and IV only D. II, and III only E. NOTA

11. Given the following system of equations, find $\frac{ABC^2}{5D}$.

A + 4B + C + 4D = 9
2A +2B – C + 2D = -6
3A – 2B + 2C + D = 4
4A + 4B +4C + 4D = 18

A. 0 B. -5 C. -25 D. The system is inconsistent. E. NOTA

12. How many solutions does the given system of equations have?

3x + 6y = 1
4x + 8y = 4

A. 0 B. 1 C. 2 D. Infinitely many E. NOTA

13. Which of the following matrices are in reduced row echelon form?

I. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ II. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ III. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ IV. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ V. $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

A. I and III only B. I and V only
C. I, III, IV and V only D. II, III, and V only E. NOTA

14. There exists an equation of the plane containing the points (1, 2, 4), (0, 1, 2), and (-1, 0, 1) in the form of Ax + By + Cz = D, where A is less than or equal to 0 and A, B, C, and D are relatively prime integers. Compute ABD.

A. -1 B. 0 C. 14 D. 28 E. NOTA

15. Find the area of the triangle whose vertices are P = (1, 2, 4), Q = (0, 1, 2), and R = (-1, 0, 1).

A. 2 B. $\sqrt{2}$ C. $\frac{\sqrt{2}}{2}$ D. $\frac{\sqrt{2}}{4}$ E. NOTA

16. What is the scalar product of the vectors <1, -2, 3> and <1, 1, 4>?

A. 15 B. -9 C. 9 D. 11 E. NOTA

17. A is an orthogonal n x n matrix. What is A^TA?

A. A^{-1} B. –A C. $-A^{-1}$ D. I_n E. NOTA

18. A is a skew symmetric n x n matrix. What is the trace of A?

A. -1 B. 0 C. 1 D. Not enough information E. NOTA

19. The following sets of points each define the vertices of a triangle. How many of these triangles have a right angle?

I. (1, 2, 3), (-2, -1, -2), (-3, 0, 1) II. (0, 1, 1), (1, 0, 0), (1, 1, 1)

III. (2, 1, 1), (-4, 2, 8), (0, 7, 0) IV. (1, 1, 5), (2, -5, -1), (5, 1, 2)

A. 0 B. 1 C. 2 D. 3 E. NOTA

20. What is the unit vector in the direction of <1, -2, 1> x <-2, 1, 2>, where x denotes the cross product?

A. $\frac{1}{\sqrt{2}}$<-1, -4/5, -3/5> B. $\frac{1}{\sqrt{50}}$<5, 4, 3> C. $\frac{1}{\sqrt{50}}$<-5, 0, 5> D. $\frac{1}{\sqrt{50}}$<3, 4, 5> E. NOTA

21. In solving the following system of equations for the solution value of y, what expression would be obtained by using Cramer's rule?

$$14x + 27y - 104z = 70$$
$$5x - 19y + 11z = 17$$
$$8x + 8y + 19z = 20$$

A. $\dfrac{\begin{vmatrix} 14 & 27 & -104 \\ 5 & -19 & 11 \\ 8 & 8 & 19 \end{vmatrix}}{\begin{vmatrix} 14 & 70 & -104 \\ 5 & 17 & 11 \\ 8 & 20 & 19 \end{vmatrix}}$ B. $\dfrac{\begin{vmatrix} 14 & 70 & -104 \\ 5 & 17 & 11 \\ 8 & 20 & 19 \end{vmatrix}}{\begin{vmatrix} 14 & 27 & -104 \\ 5 & -19 & 11 \\ 8 & 8 & 19 \end{vmatrix}}$ C. $(-1)\dfrac{\begin{vmatrix} 14 & 70 & -104 \\ 5 & 17 & 11 \\ 8 & 20 & 19 \end{vmatrix}}{\begin{vmatrix} 14 & 27 & -104 \\ 5 & -19 & 11 \\ 8 & 8 & 19 \end{vmatrix}}$ D. $\dfrac{\begin{vmatrix} 14 & -70 & -104 \\ 5 & -17 & 11 \\ 8 & -20 & 19 \end{vmatrix}}{\begin{vmatrix} 14 & 27 & -104 \\ 5 & -19 & 11 \\ 8 & 8 & 19 \end{vmatrix}}$

E. NOTA

22. An n x n matrix A has at least one eigenvalue λ. Which of the following statements must be true for the matrix $X = (A - \lambda I_n)$?

A. X is orthogonal. B. $\det(X) \neq 0$. C. X is similar to A.
D. X can be row reduced to the identity matrix. E. NOTA

23. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$. What is the product of the smallest and largest eigenvalues of A?

A. 24 B. 12 C. 6 D. 2 E. NOTA

24. Find the sum of all x which satisfy $\begin{bmatrix} x^2+7x-5 \\ x^2+5x-6 \\ x^2-6x+7 \end{bmatrix} - \begin{bmatrix} 2x+1 \\ 1-x \\ 3-x^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

A. -10 B. -5 C. 1 D. 3 E. NOTA

25. Standing on the western bank of a river that spans 5 miles across, Jane wishes to reach the eastern bank in her canoe. She can row her canoe at a speed of $6\sqrt{3}$ miles per hour, and the river flows due south with a current of 6 miles per hour. What distance, in miles, will Jane traverse if she crosses the river in as little time as possible?

A. 5 B. 10 C. $\frac{5\sqrt{3}}{3}$ D. $\frac{10\sqrt{3}}{3}$ E. NOTA

26. Vectors **a** = <1, 2, -2> and **b** = <3, 0, 1>. Which of the following vectors is linearly independent of **a** and **b**?

A. <-1, 4, -5> B. <5, -2, 4> C. <9, 6, -3> D. <6, 0, 2> E. NOTA

27. $A^{-1} = \begin{bmatrix} 1 & 3 & 1 \\ 0 & -6 & 7 \\ 3 & 1 & -2 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} -3 & 6 & -2 \\ 1 & -6 & 7 \\ 2 & 2 & 5 \end{bmatrix}$. What is the sum of the entries in $(AB)^{-1}$?

A. 12 B. 22 C. 48 D. 54 E. NOTA

28. $A^{T} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ and $B^{T} = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$. What is the product of the entries in $(AB)^{T}$?

A. 0 B. -3 C. -9 D. -27 E. NOTA

29. If $\begin{vmatrix} x & 1 & 5 \\ -2 & x & 1 \\ x & -1 & x \end{vmatrix} = 0$, what statement accurately describes the only real solution for x?

A. The solution is positive B. The solution is negative C. The solution is 0
D. The solution is not an integer E. NOTA

30. What is the cosine of the angle between the two lines given by the following parametric equations?

$$L_1 = \begin{cases} x = t + 2 \\ y = 2t \\ z = 4 - t \end{cases} \quad \text{and} \quad L_2 = \begin{cases} x = 4t - 3 \\ y = t + 3 \\ z = t - 3 \end{cases}$$

A. $\frac{2\sqrt{3}}{9}$ B. $\frac{5\sqrt{42}}{63}$ C. $\frac{5\sqrt{3}}{18}$ D. $-\frac{\sqrt{15}}{5}$ E. NOTA