Geometry – Hustle Solutions
National MAΘ 2008

#1 65π

$V = \frac{1}{3}Bh; 100\pi = \frac{1}{3} \bullet 25\pi h; h = 12$, $l = 13$, $LA = \pi rl = 5 \bullet 13 \bullet \pi = 65\pi$

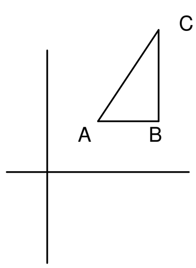
#2 $2\sqrt{15}$

From the center of the circle to the tangent segment is a right angle – that distance is 5 since it's the radius of the circle. The hypotenuse of the triangle would be the distance from the center of the circle $(-4,1)$ to the point (3,7) – which is $\sqrt{85}$. Using Pythagorean theorem, we get that the length of the tangent segment is $\sqrt{60} = 2\sqrt{15}.$

#3 $8\sqrt{3}$

Let the hexagon be named ABCDEF. One of the shorter diagonals would be $\overline{AE}$.
In triangle AEF, $m\angle F = 120$ and the other two angles are 30. Draw an altitude from F forming 2 30-60-90 triangles. The side of the hexagon is 8 making the altitude 4, and $\frac{1}{2}$ of the diagonal $4\sqrt{3}$ making the diagonal $8\sqrt{3}$.

#4 $1+2\sqrt{3}$



Point A has vertices (3,1). Point B has vertices (5,1).
Point C is (5,*a*). $m\angle A = 60$, $AB = 2$, $AC = 4$, $BC = 2\sqrt{3}$ making the *y* coordinate of point C $1+2\sqrt{3}$.

#5 36

Let a side of the smaller triangle be *x*, a side of the larger triangle would be *x* +10. The sum of the perimeters would be $6x+30$ which equals 186. $x = 26$ which makes the side of the larger triangle 36.

#6 10000

$SA = LA + 2B; 2800 = 4s \bullet 25 + 2s^2;$ solving this quadratic gives $x = 20.$ Volume is $20 \bullet 20 \bullet 25$ which is 10000.

#7 32

Since the radius of the circle is 4, the diagonal of the square is 8 and a side of the square is $4\sqrt{2}$ which makes the area of the square 32.

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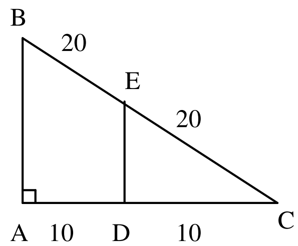
#8 $30+10\sqrt{3}$

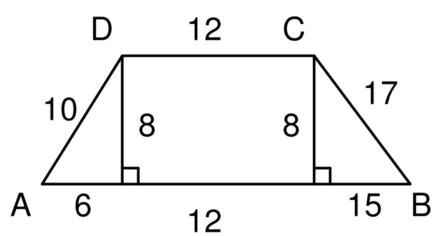


Since D and E are midpoints, $\overline{DE}$ is parallel to $\overline{AB}$ and half it's length. $BC = 2 \bullet AC$, so this is a 30-60-90 triangle making $AB = 20\sqrt{3}$ and $DE = 10\sqrt{3}$. The perimeter of CDE is $30+10\sqrt{3}$.

#9 $6\sqrt{10}+6\sqrt{5}$

Find the distance between each pair of points. $AB = \sqrt{81+9} = 3\sqrt{10}$, $BC = \sqrt{144+36} = 6\sqrt{5}$, $AC = \sqrt{9+81} = 3\sqrt{10}$ making the perimeter $6\sqrt{10}+6\sqrt{5}$.

#10 180



Using Pythagorean theorem to find the bases of the triangles. Find the area using $\frac{1}{2}h(b_1+b_2)$.

$\frac{1}{2} \bullet 8(12+33) = 180$

#11 3

Let the radius of circle A be *x*. That makes the radius of circle B $10-x$ and the radius of circle C $14-x$. We know $BC = 18$ which is also the sum of the radii of circle B and C. So we have the equation $18 = 10-x+14-x$. $x = 3$ which is the radius of Circle A.

#12 $4\sqrt{5}$

The midpoint of the hypotenuse of a right triangle is equidistant from the vertices of the triangle. Therefore, the median is 6 and the hypotenuse would be $2 \bullet 6 = 12$. Using Pythag to find CB makes CB = $4\sqrt{5}$.

#13 $\frac{45}{2}$

Use the formula $\frac{|60h-11m|}{2} = \frac{|60 \bullet 2 - 15 \bullet 11|}{2} = \frac{45}{2}$.

#14 rhombus

The four triangles formed by joining the midpoints are congruent by SAS making all four sides of the quadrilateral congruent. Opposite sides being congruent makes it a parallelogram, and all four sides congruent makes it a rhombus.

#15 $\frac{3}{2}\pi$

$\frac{30}{360} \bullet 18\pi = \frac{3}{2}\pi$.

#16 9

$\frac{\cancel{1}}{\cancel{2}} 18\cancel{h} = \frac{\cancel{1}}{\cancel{2}}\cancel{h}\left(b_1 + b_2\right)$; making the sum of the bases 18 which makes the median 9.

#17 108

Use the formula $d = \sqrt{l^2 + w^2 + h^2}$ to find the height.

$11 = \sqrt{4 + 36 + h^2}; 121 = 40 + h^2; 81 = h^2; h = 9$. $V = 2 \bullet 6 \bullet 9$.

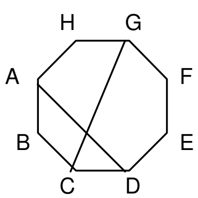
#18 12

The ratio of the areas is 96:6 which simplifies to 16:1. That makes the ratio of the sides and perimeters 4:1. Set up the proportion $\frac{4}{1} = \frac{48}{x}, x = 12$.

#19 $\sqrt{3}$

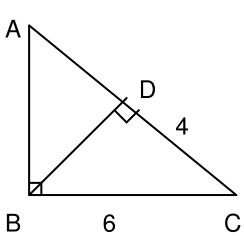
$\sqrt{3} = \frac{s^2\sqrt{3}}{4}; s^2 = 4, s = 2.$ Drawing the altitude, which is opposite a $60°$ angles in a 30-60-90 where the hypotenuse is 2, makes the altitude $\sqrt{3}$.

#20 $\frac{135}{2}$



$m\angle ADE = 90$ making $\angle ADC = 135 - 90 = 45$, since the measure of one interior angle of an octagon is 135. $m\angle DCG = \frac{135}{2}$ since CG is the longest diagonal it bisects the angles of the hexagon. Add these and subtract from 180 to find the acute angle is $\frac{135}{2}$.

#21 $3\sqrt{5}$



Use the similarity between the right triangles to find AD.

$6 = \sqrt{4 \bullet AD}$, $AD = 5$. $AB = \sqrt{5 \bullet 9}$ which makes $AB = 3\sqrt{5}$.

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#22 $2\sqrt{85}\pi$

The segment from the center to the chord bisects the chord and is perpendicular to the chord giving a right triangle with legs 6 and 7. Use Pythagorean theorem to find the radius is $\sqrt{85}$ making the circumference $2\sqrt{85}\pi$.

#23 13

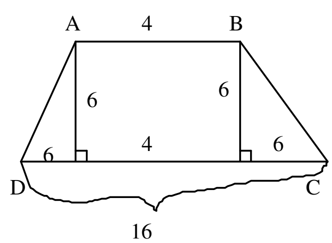
$A + B = 90, 180 - A - (180 - B) = 64.$ Solving this system makes $B = 77, A = 13.$

#24 $24\sqrt{3}$

The longest diagonal is 8. If all diagonals were drawn we would have 6 equilateral triangles with a side of 4. To find the area of the hexagon find the area of one triangle and multiply by 6.

$6 \bullet \frac{16\sqrt{3}}{4} = 24\sqrt{3}$.

#25 90



See diagram for lengths of segments. Since the triangles have legs of 6, $m\angle D = 45$ which is the lower base angle, and the upper base angle would be 135. The difference is 90.