Trig – Hustle
National MAΘ 2008

1. $\frac{3}{\pi}$
2. 7
3. 66
4. $12+4\sqrt{7}$
5. 5
6. $-63/65$
7. $\pi/3$
8. -3
9. $x = \frac{-\pi}{6}, \frac{-5\pi}{6}$
10. -1
11. 1
12. $\frac{x^2}{\sqrt{1-x^2}}$
13. $\sqrt{3}/2$
14. 0.3
15. $24\sqrt{2}$
16. 5
17. 24
18. 60
19. 135°
20. 7 meters
21. x/4
22. -1
23. $12\sqrt{2}$
24. 12
25. $\sqrt{3}/2$

Solutions:

1. Phase shift = -c/b = -(-1)/(π/3) = 3/π

2. $2(\frac{1-\sec^2 x}{\sec^2 x})\frac{2}{\csc x} = 2(\cos^2 x - 1)2\sin x$

$4\sin^2 x \sin x = 4\sin^3 x$

$4+3=7$

3. $30+.5B+B+60=180$

$1.5B=90$ Equilateral so all sides are 6. 60+6 = 66

$B=60$

$A=60$

$C=60$

4. $f^2 = 4^2+8^2-2(4)(8)\cos 120$

$f^2 = 112$

$f=\sqrt{112}$

$p=4+8+\sqrt{112}=12+4\sqrt{7}$

5. $(\cos 5x\cos 2x-\sin 5x\sin 2x)-(\cos 5x\cos 2x+\sin 5x\sin 2x)$

$=-2\sin 5x\sin 2x$

$-2+5+2=5$

6. $\sin x\cos y+\cos x\sin y$

$\left(\frac{12}{15}\right)\left(\frac{-12}{13}\right)+\left(\frac{-9}{15}\right)\left(\frac{5}{13}\right)$

$\frac{-48}{65}-\frac{15}{65}=\frac{-63}{65}$

7. $Arc\sin(\cos(Arc\tan(\frac{-1}{\sqrt{3}}))) = ?$

$Arc\tan(\frac{-1}{\sqrt{3}}) = \frac{-\pi}{6}$

$\cos\frac{-\pi}{6} = \frac{\sqrt{3}}{2}$

$Arc\sin\frac{\sqrt{3}}{2} = \frac{\pi}{3}$

8. $\cos^2 x(1-\sec^2 x) = \cos^2 x(-\tan^2 x) = -\sin^2 x$

$-1-2=-3$

9. $(2\sin x+1)(\sin x+3)$

$\sin x = \frac{-1}{2}, -3$

$\sin x = \frac{-1}{2}, x = \frac{-\pi}{6}, \frac{-5\pi}{6}$

10. $y = -2(\sin\frac{1}{(4x-\pi)})+1$

$y=-2+1=-1$

$\cos y = x^2 - 2$

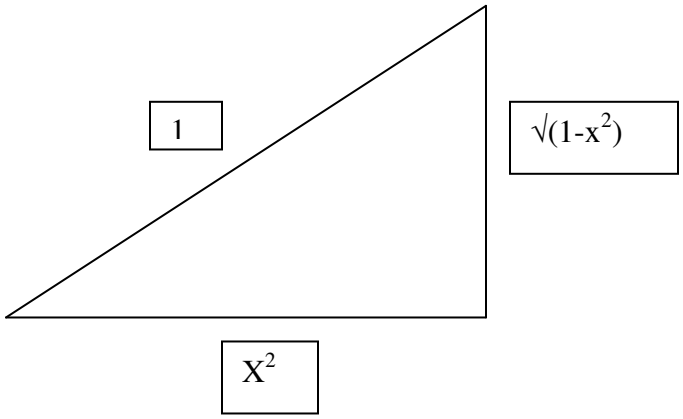
$\cos 2y = 2\cos^2 y - 1$

11. $= 2\left(x^2 - 2\right)^2 - 1$

$\cos 2y = 2x^4 - 8x^2 + 7$

$Find: A + B + C = 1$

12. $\cot\Theta = \dfrac{x^2}{\sqrt{1-x^2}}$



13. $2\sin y(1 - 2\cos y) = 0$

$\sin y = 0, y = 0$

$\cos y = 1/2$

$y = \dfrac{\pi}{3}$

$\sin 2y = \sin\dfrac{2\pi}{3} = \dfrac{\sqrt{3}}{2}$

$y = -3\tan(\dfrac{2\pi x}{3} - 1) + 5$

14. $period = \dfrac{\pi}{B} = \dfrac{\pi}{\frac{2\pi}{3}} = 1.5$

$vertshift = 5$

$1.5/5 = 0.3$

$A = \dfrac{1}{2}ab\sin C$

15. $A = \dfrac{1}{2}(8)(12)\sin 45$

$A = \dfrac{1}{2}(8)(12)\dfrac{\sqrt{2}}{2} = 24\sqrt{2}$

16. $\pm\sqrt{\dfrac{1+\cos\frac{5\pi}{6}}{2}} = \sqrt{\dfrac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\dfrac{\frac{2-\sqrt{3}}{2}}{2}} = \sqrt{\dfrac{\frac{2-\sqrt{3}}{4}}{1}} = \dfrac{\sqrt{2-\sqrt{3}}}{2}$

$2 + 3 = 5$

17. $4(6)cis(\dfrac{3\pi}{4} + \dfrac{5\pi}{4})$

$24cis2\pi = 24(\cos 2\pi + i\sin 2\pi) = 24(1 + 0i) = 24$

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18. Angle A must be a right angle, making ABC a right triangle. Angle B can only be 60°.
19. Eric ends up 15 miles east and 15 miles south. Isosceles right triangle means his angle is 45 degrees. Bearing is 135 degrees.
20. Jill makes a 9-12-15 triangle with her kite. Jack makes a 12-16-20 triangle. 16-9=7
21. adj = x, opp = 4, hyp = $\sqrt{(x^2+16)}$. Cotangent=adj/opp = x / 4

$\cos\pi + \sin\pi = -1$

22. $\cos 2\pi + \sin 2\pi = 1$

...

$\cos 15\pi + \sin 15\pi = -1$

$\frac{\sin 30}{12} = \frac{\sin 45}{y}$

23. $0.5y = \frac{\sqrt{2}}{2}(12)$

$y = 12\sqrt{2}$

$\frac{\sin 105}{z} = \frac{\sin 30}{12}$

$\sin 105 = \sin(60+45) = \sin 60\cos 45 + \cos 60\sin 45$

24. $= (\frac{\sqrt{3}}{2})(\frac{\sqrt{2}}{2}) + (\frac{1}{2})(\frac{\sqrt{2}}{2}) = \frac{\sqrt{6}+\sqrt{2}}{4}$

$12(\frac{\sqrt{6}+\sqrt{2}}{4}) = 0.5z$

$z = 6\sqrt{6} + 6\sqrt{2}$

$6+6=12$

25. $\sqrt{3}/2$ from unit circle (equivalent to 30 degrees)