For **all** questions, choice E is “NOTA” (none of the above).

1. Find the product of the least common multiple and greatest common factor of 56 and 72.

A. 4032 B. 3888 C. 2592 D. 1296 E. NOTA

2. For how many positive triples (x, y, z) does $x^4 + y^4 = z^4$?

A. None B. One C. Two D. Infinite E. NOTA

3. What is the probability that a randomly selected factor of 49,504 is even?

A. $1/2$ B. $4/5$ C. $5/6$ D. 1 E. NOTA

4. How many positive solutions does $x = 4 \bmod 7$ for $x < 755$?

A. 107 B. 108 C. 109 D. Infinite E. NOTA

5. Find the sum of digits of the base 16 expression of 1101010010001001_2.

A. 7_{16} B. 12_{16} C. 18_{16} D. 22_{16} E. NOTA

6. Which of the following is **not** yet known to be true of all pairs of twin primes?

A. Both elements are odd
B. Both elements are prime
C. There are an infinite number of them.
D. The two elements are relatively prime
E. NOTA

7. If y has 25 factors, y^2 can have z factors. Find the sum of all possible values of z.

A. 52 B. 130 C. 154 D. Infinity E. NOTA

8. In a 97 person single elimination tournament with byes, z games must be played to determine a winner. (A bye doesn’t count as a game) Find the sum of the digits of z.

A. 15 B. 16 C. 17 D. 18 E. NOTA

9. For how many positive integers b is $94_b > 124_b$ a valid statement in base b?

A. 0 B. 6 C. 7 D. 8 E. NOTA

10. Which of the following is closest to the probability that a randomly chosen nonnegative integer less than 1584 is relatively prime to 1584?

A. 0.2 B. 0.4 C. 0.6 D. 0.8 E. NOTA

11. Which of the following values of y satisfies $3^{99} \equiv y \bmod 64$?

A. 3 B. 9 C. 73 D. 81 E. NOTA

12. Find the sum of the factors of 1920.

A. 3048 B. 6120 C. 24,384 D. 48,960 E. NOTA

13. In how many zeroes does 123! end?

A. 16 B. 22 C. 24 D. 28 E. NOTA

14. How many nonnegative integer solutions does $5x + 8y = 80360$ have?

A. 2008 B. 2010 C. 5040 D. 8064 E. NOTA

15. Find the sum of the 5 smallest positive x such that $x \equiv 5 \bmod 7$, $x \equiv 9 \bmod 11$, $x \equiv 0 \bmod 2$.

A. 770 B. 2300 C. 2308 D. 2310 E. NOTA

16. Who proved that the set of primes is infinite?

A. Euclid B. Gauss C. Euler D. Fermat E. NOTA

17. What is the largest integer that **cannot** be written in the form $9x+9y+16z$ where x and y are multiples of 9 and z is a multiple of 16?

A. 25 B. 110 C. 119 D. 145 E. NOTA

18. How many distinct prime factors does 50! have?

A. 13 B. 14 C. 15 D. 50 E. NOTA

19. What is the sum of the two smallest perfect numbers?

A. 30 B. 13 C. 20 D. 5 E. NOTA

20. How many ordered pairs of integers (x, y) satisfy the equation $x^2 y = 6!$?

A. 2 B. 5 C. 6 D. 10 E. NOTA

21. Which of the following functions does not have the property that $f(a)f(b) = f(ab)$ if *a* and *b* are relatively prime?

A. $f(n) =$ the number of factors of n
B. $f(n) =$ the sum of the factors of n
C. $f(n) = \gcd(n,162)$
D. $f(n) = lcm(n,162)$
E. NOTA

22. How many positive values of k satisfy the following condition?
For any integer x, at least one of x, x^2-1, x^2+1 must be divisible by k.

A. 0 B. 1 C. 2 D. 4 E. NOTA

23. If $5xy+8z \equiv 4 \bmod 18$, find $(2z-xy) \bmod 9$.

A. 0 B. 1 C. 2 D. 3 E. NOTA

24. How many even two digit numbers have an odd number of distinct factors?

A. 4 B. 5 C. 8 D. 10 E. NOTA

25. What is sum of the two smallest positive values of n such that the sum of the first n positive cubes is divisible by 8?

A. 5 B. 10 C. 15 D. 20 E. NOTA

26. If $f(x) \equiv 2x \bmod 17$ and $0 \le f(x) < 17$, find $f(300)$.

A. 5 B. 8 C. 9 D. 16 E. NOTA

27. How many of the first 120 elements of the Fibonacci sequence starting with $1,1,...$ are divisible by 4?

A. 10 B. 20 C. 30 D. 40 E. NOTA

28. How many positive integers less than 108 are relatively prime to both 24 and 27?

A. 18 B. 36 C. 54 D. 72 E. NOTA

29. How many positive integers less than 100 cannot be expressed in the form of $3x+12y$, where x and y are integers?

A. 13 B. 20 C. 66 D. 67 E. NOTA

30. What mathematician is famed for his discovery of the closed form expression of triangular numbers? Legend has it that he was a merely in grade school when his teacher asked him to sum the first 100 numbers and only to be astonished how quickly this young man could do it.

A. Blaise Pascal
B. Leonardo Fibonacci
C. Leonhard Euler
D. Carl Friedrich Gauss
E. NOTA