1. **A.** There are 3! ways of ordering the topics and 3!, 4! and 3! ways of ordering the individual topics. \((3!)(3!)(4!)(3!))

2. **B.** There are 9x10x10x10x10=90000 5-digit numbers, half of which (45000) are even.

3. **C.** All digits must be distinct and any set of 5-digits can be put in order in only one way so the answer is \(\binom{10}{5} = 252\)

4. **C.** \(240 = 2^4 \cdot 3 \cdot 5\), so in factors there are 5 choices for a power of 2 (0,1,2,3,4), 2 choices for a power of 3 and 2 for 5. \(5 \cdot 2 \cdot 2 = 20\).

5. **B.** From any given vertex, one can draw 3 rectangles (ABEF, ADEH and ACEG) multiply by 8 vertices and divide by the 4 vertices in each rectangle \(3 \cdot 8 / 4 = 6\).

6. **C.** Three distinct triangles can be drawn with a given vertex at the right angle. \(3 \cdot 8 = 24\).

7. **B.** Ten letters are to be permuted but the two ‘A’s’ can appear in any order without distinction. \(10!/2 = 5(9!)\).

8. **B.** Using at most one of the A’s there are 9x8x7=504 ways. Using both A’s, there are 8 letters to choose for the third letter and 3 ways to permute them, \(3 \cdot 8 = 24\). Total 504+24=528.

9. **A.** Of the 8 steps, 4 must be to the right and 4 up and any combination of these will work. So \(\binom{8}{4} = 70\)

10. **D.** There are two possibilities: 5 steps to the right, 1 to the left and 4 up or 4 steps to the right, 5 up and 1 down. So the answer is \(\frac{10!}{4!4!} = 2520\)

11. **D.** There are four possibilities. The first has 2 steps to the right, 2 up and 2 diagonally up-right = \(6!/(2!2!2!) = 15\). The others include 4 diagonal steps up-right and either an up-down, \(6!/(4!1!1!) = 30\); left-right, 30; or diagonal up and back, \(6!/(5!1!) = 6\) for a total of 81.

12. **E.** Add three balloons to the allotment so that everyone will get at least one. Lining up the 13 balloons, choose 2 of the 12 gaps between balloons. This corresponds to a division of the balloons. \(\binom{12}{2} = 66\)

13. **D.** Solve (as in the previous problem) the division of each color and then multiply. \(\binom{5}{2} \binom{5}{2} = (10)(10)(15) = 1500\)

14. **A.** This is the so called “hockey-stick” formula from Pascal’s Triangle. The sum \(
\begin{align*}
\binom{12}{9} &= \frac{12\cdot11\cdot10}{3\cdot2} \\
&= 220
\end{align*}
\)

15. **E.** The sum of the exponents of any term will be 6, so this problem is equivalent to dividing 6 factors among 3 variables. \(\binom{8}{2} = 28\)

16. **B.** There are 5C2=10 ways of picking the two teachers that must get their seat right. There are only two ways the other 3 can sit so that they’re all in the wrong seat (BCA, CAB). \(10 \cdot 2 = 20\)

17. **A.** The last 4 flips must be THHH. There are 16 possibilities for the first 4 flips but HHHH, HHHT and THHH all result in a sequence of three heads. \(16 – 3 = 13\).

18. **B.** There are two possibilities for the first person – inside or outside. The other 7 can be place in \(7!\) ways. \(2 \times 7! = 10080\).
19. C. There are 7 kinds of food and he must choose 5 of them. \( \binom{7}{5} = 21 \).

20. B. There are 3 ways to get the 5 servings: 5 different veg. (7C5); 2 servings of 1 kind and 1 of 3 others (7C1)(6C3) or 2 of 2 kinds and 1 of a third (7C2)(5C1). Adding, you get 266.

21. E. The digit 5 must be picked and of the digits 1, 2, 3 and 4 you pick two and also two from 6, 7, 8 and 9. Once picked there are 5! = 120 numbers that can be made. 6x6x120 = 4320.

22. C. This is the partition problem. In this case, the easiest way to solve it is via a list: 9+2+1, 8+3+1, 7+4+1, 7+3+2, 6+5+1, 6+4+2, 5+4+3.

23. B. There are 2 partitions of 5 that will work: 2+2+1 means that of the 4 boxes, we choose 2 to get balls and of the remaining 2 boxes, I choose 1 to get a ball. \( \binom{4}{2} \times \binom{2}{1} = 6 \times 2 = 12 \). The other partition is 2+1+1+1 and there are 4 ways of choosing the box that gets 2 balls. 12+4=16.

24. C. Treat “ABC” as 1 big letter. There are 24 ways of permuting ABC, A, B, and C. However, two are indistinguishable: ABC A B C and A B C ABC leaving 23.

25. B. There are 1260 ways total \( \frac{7!}{(2!)^2} \) with 180 starting with ‘A’, 360 starting with ‘E’, 180 starting with ‘L’, 180 with ‘S’ and 360 beginning with ‘T’. The 900\(^{th}\) word will be the last word starting with ‘S’ or STTLEEA.

26. E. There are 6! ways of rolling 5 different numbers and 5! ways of each straight. 6! – 2x5! = 480.

27. A. A must be in the subset and B cannot be. C can either be in the subset of not (2 possibilities) – same for D, E, F and G. \( 2^5 = 32 \).

28. A. If a double is drawn (7 ways) it matches with 6 others. 6x7 = 42. If something other than a double is drawn (21 ways) then it matches with 12 others. 21x12 = 252. But each pair was counted twice, so the answer is \( (42+252)/2 = 147 \).

29. B. One could start with 1 dog, 1 cat, notice that there is only 1 permutation, CD or 1/2 the possible permutations. With 2 each there are 2, (CCDD and CDCC) or 1/3 the possibilities; etc. The answer is then \( \frac{1}{6} \binom{10}{5} = 42 \). OR – consider a sequence for which there are more dogs than cats at some point \( n=2k+1 \) (at 2k there are an equal number). In this sub-sequence, switch the k+1 D’s to C’s and vice versa getting a total sequence of 10 with 4 D’s and 6 C’s. On the other hand, take any sequence of 6 C’s and 4 D’s and stop when there is one more C than D (this must happen by the 9\(^{th}\) letter). Now reverse the C’s and D’s again and you get a sequence of 5 each that has more Dogs than Cats at some point. The number of good permutations is then \( 10C5 – 10C6 = 42 \).

30. D. The number of girls is 2, 3 or 4 and boys 2, 1 or 0 respectively. \( (4C2)(6C2) + (4C3)(6C1) + (4C4)(6C0) = (6)(15)+(4)(6)+(1)(1) = 115 \)