Proofs Test National MAO Test 2008

NOTA stands for None of the Above

1. At 1:00 pm on October 25, 2000, a dog falls asleep. It sleeps for 5 hours, then is awake for 5 hours, then sleeps for 5 hours, etc. At 1:17 pm on the same day, a cat falls asleep. It sleeps for 2 hours, then is awake for 2 hours, then sleeps for 2 hours, etc. Determine the largest integer H so that in every unbroken 24 hour period in the year 2001 there must be a total of at least H hours during which both animals will be asleep.

   a. 4 b. 5 c. 6 d. 7 e. NOTA

2. Suppose you are given the following three statements:
   I. No kitten that loves fish has green eyes. II. All kittens with whiskers love fish.
   III. No kitten has a tail unless it has whiskers.
   Which of the following statements is a valid conclusion?

   a. No kitten with green eyes has a tail b. All kittens with tails have green eyes
   c. No kitten that loves fish has a tail d. All kittens that have tails have no whiskers
   e. NOTA

3. A playing field is filled with 2001 Trolls, 2002 Griffins, and 2003 Dragons. Whenever two animals of different species shake hands, they both instantly disappear and are replaced by an animal of the third species. This game continues for some time, until there is only one species of animal left in the field. Which of the following is a possible end position of such a game?

   a. Exactly 5 Trolls remain b. Exactly 25 Griffins remain c. Exactly 9 Trolls remain
   d. Exactly 4 Dragons remain e. NOTA

4. Given the conditional statement: "If two lines are parallel, then the lines do not intersect." Which of the following conditionals is equivalent to the given statement?

   a. "If two lines do not intersect, then the lines are parallel."
   b. "If two lines intersect, then the lines are not parallel."
   c. "If two lines are not parallel, then the lines intersect."
   d. "If two lines are not parallel, then the lines do not intersect."
   e. NOTA

5. Given the statement "If a pentagon is regular, then it is equiangular," which of the following is true?

   a. only the conditional is true
   b. only the conditional and contrapositive are true
   c. only the conditional, converse, and inverse are true
   d. the conditional, converse, inverse, and contrapositive are all true
   e. NOTA

6. The positive integer n is said to have "Property T" if the integers 1 through $3n$ can be partitioned into n mutually disjoint sets, $S_1, \ldots S_n$ such that each S_i contains 3 integers and there is an integer m such that the sum of the integers in S_i is m^2 for each i. Which of the following statements is correct?

   a. No positive integer has Property T

   b. There is exactly one positive integer that has Property T

   c. There is more than one positive integer that has Property T, but the set of positive integers with Property T is finite

   d. Every positive integer has Property T

   e. NOTA

7. For $n = 1,2, \ldots,$ let $T_n = 1 + 2 + \ldots + n$. Which of the following statements is correct?

   a. There is no value of n for which T_n is a positive power of 2. (A positive power of two is an integer of the form 2^k where k is a positive integer.)
   b. There is exactly one value of n for which T_n is a positive power of 2.
   c. There are exactly two values of n for which T_n is a positive power of 2.
   d. There are more than two but finitely many values of n for which T_n is a positive power of 2.
   e. NOTA

8. Where $A = 1$, $B = 2$,...$Z = 26$, we define $A \sim B = 2$, $A \sim B \sim C = 6$, $A \sim C \sim L = 36$, and $A \sim B \sim C \sim ... \sim Z = 26!$. What set of three letters equals 1001 under the ~ operation?

   a. C, K, W b. C, Q, S c. G, I, M d. G, K, M e. NOTA

9. A specific principle used in proofs states that given kn + 1 objects which are placed in n boxes, then there must be some box with k + 1 objects. What is this principle called?

   a. Box Principle b. Pigeonhole Principle c. Basket Principle
   d. Inductive Reasoning Principle e. NOTA

10. The first major theorem that was proved using a computer was:

   a. The Four Color Theorem b. Fermat's Theorem c. Euler's Theorem
   d. De Moivre's Theorem e. NOTA

11. To **properly** prove a theorem using mathematical induction, one would start the proof by:

   a. Contradicting the statement of what one is trying to prove.
   b. Writing down the given information in the left column of a 2-column proof.
   c. Trying to show that the statement one is trying to prove works for the number 1.
   d. Rewriting what one is trying to prove in the form of a series.
   e. NOTA

12. Assume you have a chocolate bar consisting, as usual, of n squares arranged in a rectangular pattern. Your task is to split the bar into small squares (always breaking along the lines between the squares) with a minimum number of breaks. How many will it take?

   a. n b. n – 1 c. n – 2 d. ½ n e. NOTA

13. There is obviously a serious flaw in the following proof. In what line is the flaw?

   1. $(n+1)^2 = n^2+2n+1$
   2. $(n+1)^2-(2n+1) = n^2$

3. $(n+1)^2-(2n+1)-n(2n+1) = n^2-n(2n+1)$
4. $(n+1)^2-(n+1)(2n+1) = n^2-n(2n+1)$
5. $(n+1)^2-(n+1)(2n+1)+(2n+1)^2/4 = n^2-n(2n+1)+(2n+1)^2/4$
6. $[(n+1)-(2n+1)/2]^2 = [n-(2n+1)/2]^2$
7. $(n+1)-(2n+1)/2 = n-(2n+1)/2$
8. $n+1 = n$
9. $1 = 0$

a. Line 4 b. Line 5 c. Line 6 d. Line 7 e. NOTA

14. A telephone number has the form *ABC – DEF - GHIJ*, where each letter represents a different digit. The digits in each part of the number are in decreasing order; that is, $A > B > C$, $D > E > F$, and $G > H > I > J$. Furthermore, *D*, *E*, and *F* are consecutive even digits; *G*, *H*, *I*, and *J* are consecutive odd digits; and $A + B + C = 9$. Find *A*.

a. 4 b. 5 c. 6 d. 7 e. NOTA

15. Barry pours four ounces of coffee into an eight-ounce cup and four ounces of cream into a second cup of the same size. He then transfers half the coffee from the first cup to the second and, after stirring thoroughly, transfers half the liquid in the second cup back to the first. What fraction of the liquid in the first cup is now cream?

a. 1/4 b. 1/3 c. 3/8 d. 2/5 e. NOTA