Theta Radicals Solutions Nationals 2008

1. $6-x=x^2 \rightarrow x=2,-3 \rightarrow \sqrt{6-(-3)} \neq -3 \rightarrow \boxed{\{2\}}$ **A**.

2. $\sqrt{3-x}=x \rightarrow x^2+x-3 \rightarrow x=\boxed{\frac{\sqrt{13}-1}{2}}$ **D**.

3. $2x^2-7=9-6x+x^2 \rightarrow (x+8)(x-2)=0 \rightarrow \boxed{\{-8,2\}}$ **C**.

4. $2x+9+x+1+2\sqrt{2x^2+11x+9}=x+4 \rightarrow$
$2x+6=-2\sqrt{2x^2+11x+9} \rightarrow x^2+6x+9=2x^2+11x+9 \rightarrow x=-5,0$ but
$\sqrt{2(-5)+9}+\sqrt{(-5)+1} \neq \sqrt{(-5)+4} \rightarrow \boxed{0}$ **B**.

5. $x^4-4x^3+6x^2-4x+1=x^4-4x^3+x+5 \rightarrow (2x+1)(3x-4)=0 \rightarrow x \neq -0.5 \rightarrow x=\boxed{\frac{4}{3}}$ **E**

6. $l=\frac{5}{1-\frac{\sqrt{2}}{4}}+\frac{5\cdot\frac{\sqrt{2}}{4}}{1-\frac{\sqrt{2}}{4}}=\frac{40+10\sqrt{2}}{7}+\frac{10\sqrt{2}+5}{7}=\boxed{\frac{45+20\sqrt{2}}{7}}$ **D**.

7. Compare $\left(\sqrt{2}\right)^6$ with $\left(\sqrt[3]{3}\right)^6$ and see that $\left(\sqrt{2}\right)^6=8<9=\left(\sqrt[3]{3}\right)^6 \rightarrow \boxed{\sqrt[3]{3}}$ **B**.

8. $8+\sqrt{60}=a+b+2\sqrt{ab} \rightarrow a=5,b=3 \rightarrow a-b=\boxed{2}$ **D**.

9. $3-x^2 \geq 0 \rightarrow x^2 \leq 3 \rightarrow |x| \leq \sqrt{3} \rightarrow \boxed{\left\{x \middle| x \in \left[-\sqrt{3},\sqrt{3}\right]\right\}}$ **D**.

10. The max is at x=0 so $f(0)=$ $f(0)=\sqrt{3}$ min is $x=\pm\sqrt{3} \rightarrow f\left(\sqrt{3}\right)=f\left(-\sqrt{3}\right)=0 \rightarrow \boxed{\left\{y \middle| y \in \left[0,\sqrt{3}\right]\right\}}$ **B**.

11. Let log x = y $\frac{1}{2}y=\sqrt{y} \rightarrow y=4,0 \rightarrow \log x \neq 0 \rightarrow \log x=4 \rightarrow x=\boxed{10000}$ **D**.

12. $d=\sqrt{(19-3)^2+(5+2)^2}=\boxed{\sqrt{305}}$ **C**.

13. A isn't a number, B is 2, C is 1/3 and D is. $\boxed{\pi}$ **D**

14. $\frac{1}{\sqrt[3]{a^2}-\sqrt[3]{ab^2}+\sqrt[3]{b^4}}\cdot\frac{\sqrt[3]{a}+\sqrt[3]{b^2}}{\sqrt[3]{a}+\sqrt[3]{b^2}}=\boxed{\frac{\sqrt[3]{a}+\sqrt[3]{b^2}}{a+b^2}}$ **E**.

15. $\sqrt{\left(1-\sqrt{3}\right)^2+\left(\sqrt{3}-1\right)^2}=\sqrt{8-4\sqrt{3}}=\boxed{\sqrt{6}-\sqrt{2}}$ **A**.

16.
$\sqrt{©^2-©-12}^{\left(©^2-©-2\right)}=\sqrt{©^2-5©+4}^{\left(©^2+4©+3\right)} \rightarrow \left((c-4)(c+3)\right)^{0.5(c-2)(c+1)}=\left((c-4)(c-1)\right)^{0.5(c+1)(c+3)} \rightarrow$
$c=-1(1=1), c=4(0=0) \rightarrow \boxed{\{-1,4\}}$ **A**.

17. Set $\sqrt{y-1}=y-3 \rightarrow y^2-7y+10 \rightarrow y=2,5$ but at y=1 it is still $0>-2$. $\boxed{[1,5)}$ **A**.

18. $0.25<0.5<\sqrt{5}-\sqrt{3}$ and $\sqrt{3}+\sqrt{5}<4<8$ **E**.

19. $x-2 \geq \sqrt{x+4} \cup x-2 \leq -\sqrt{x-4}$ find the intersection points
$(x-2)^2=x+4 \rightarrow x(x-5)=0 \rightarrow \sqrt{x+4} \rightarrow \boxed{[-4,0] \cup [5,\infty)}$ **C**.

20. $\frac{2}{\sqrt{3-x}}=x \rightarrow x^3-3x^2+4=0 \rightarrow (x-2)^2(x+1)=0 \rightarrow x \neq -1$ so $x=\boxed{2}$ **B**.

21. $\begin{vmatrix} 7 & 6 \\ 2 & 3 \end{vmatrix} = 21-12 = 9$, so $\sqrt{9} = 3$ **D.**

22. $x^2\sqrt{x} - 2x^2 - 3x\sqrt{x} + 6x = 0 \rightarrow x\left(\sqrt{x}-2\right)(x-3) = 0 \rightarrow 0+3+4 = \boxed{7}$ **C.**

23. $\sqrt{3} \approx 1.73205... \rightarrow a_3 = \boxed{2}$ **C.**

24. $x+13 = (x-7)^2 \rightarrow (x-3)(x-12) = 0 \rightarrow \sqrt{3+13} \neq 3-7 \rightarrow \boxed{\{12\}}$ **B.**

25. $\sqrt{3-y} = 4 + \sqrt{y+7} \rightarrow 3-y = 16+y+7+8\sqrt{y+7} \rightarrow y+10 = -4\sqrt{y+7} \rightarrow$
$y^2 + 20y + 100 = 16y + 112 \rightarrow (y+6)(y-2) = 0 \rightarrow \sqrt{9} - \sqrt{1} \neq 4 \rightarrow \sqrt{1} - \sqrt{9} \neq 4 \rightarrow \boxed{\varnothing}$ **E.**

26. rewrite as $\dfrac{\sqrt{(x+7)(x-2)}}{\sqrt{(2x+1)(x-3)}} \cdot \dfrac{\sqrt{(x-3)(x-2)}}{\sqrt{(x+7)(x-1)}} \cdot \dfrac{\sqrt{(x-1)(x+3)}}{\sqrt{(x+3)(2x+1)}} = \dfrac{x-2}{2x+1}$ at x = 10 $\boxed{\dfrac{8}{21}}$ **B.**

27. $RQ_{2006} = \dfrac{\sqrt{36}}{1.2^3} \cdot k = 250 \rightarrow k = 72 \rightarrow RQ_{2008} = \dfrac{\sqrt{49}}{\frac{1}{3^3}} \cdot 72 = \boxed{13608}$ **A.**

28. $\dfrac{2}{1+\sqrt{3}} \cdot \dfrac{1-\sqrt{3}}{1-\sqrt{3}} = \dfrac{2-2\sqrt{3}}{-2} = \boxed{\sqrt{3}-1}$ **B.**

29. $\sqrt{1136} \approx 33.7046 \rightarrow \boxed{34}$ **D.**

30. $\sqrt{2}^x = x \rightarrow x = \boxed{2}$ **B.**