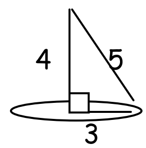
Solutions:

1. C	7. D	13. B	19. E	25. C
2. A	8. B	14. A	20. B	26. B
3. B	9. D	15. A	21. B	27. C
4. D	10. D	16. C	22. A	28. D
5. A	11. C	17. C	23. D	29. C
6. A	12. D	18. B	24. D	30. D

1. **C.** The great circle has area πr^2 and the sphere has area $4\pi r^2$. 4(6)=24.

2. **A**. $V=\frac{1}{3}\pi 3^2(4)$



3. **B**. The height will be equal to the diameter. Start with $\frac{4}{3}\pi r^3=36\pi$ to get r=3. The cylinder is $\pi(3^2)(6)$.

4. **D**. $\frac{1}{4}+\frac{1}{8}=\frac{3}{8}$ So $\frac{5}{8}(height)=20$ and h=32.. Volume = $\pi(2^2)(32)$ = 128π.

5. **A**. The water level rises from 10 to 12 ft so the displacement of the water is volume of the sphere: $\pi r^2 h=\pi(36)(2)=72\pi$.

6. **A**. $\sqrt{8^2+7^2+z^2}<12$ for z positive, gives $z^2<31$. The greatest possible integer for z is 5.

7. **D**. The surface area is area of the base plus $\frac{1}{2}l(p)$. Since the area of the base is 1, we have $\frac{1}{2}(slant)4=\sqrt{2}$. Use the Pythagorean Theorem to get $\sqrt{\left(\frac{1}{2}\right)^2+x^2}$ for our slant height. Setting this equal to $\frac{\sqrt{2}}{2}$ from the previous equation, we solve to get $x=\frac{1}{2}$

8. **B**. The edges have cubes with two or more pink faces. The faces of the large cube are 4 by 4, so the one-pink-face cubes are 2 by 2. Six faces gives 4(6)=24 little cubes and 24/64 reduces to 3/8.

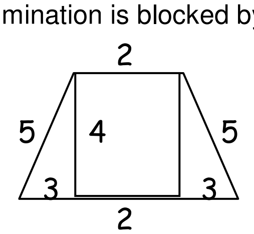
9. **D**. Cavalieri's Principle: If, in two solids of equal altitude, the sections made by planes parallel to and at the same distance from their respective bases are always equal, then the volumes of the two solids are equal.

10. **D**. $\frac{1}{3}\pi(25)(12)=\pi(64)h$ gives 25/16.

11. **C**. Euler's Formula: F+V-E=2. 40+V-54=2.

12. **D**. 1/8 of the sphere of illumination is blocked by the corner of the building so $\frac{7}{8}\bullet\frac{4}{3}\pi(2^2)=\frac{28\pi}{3}$

13. **B**. SA=ph+2B. p=20, so SA= 20(10)+2B

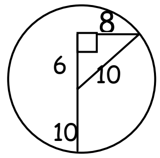


and since B= $\frac{1}{2}(4)(2+8)=20$, SA=240.

14. **A**. Use the Pythagorean Th: $\left(\sqrt{13}\right)^2+a^2=5^2$ for a=apothem of the base. This gives $a=2\sqrt{3}$ and in a

hexagon, this makes the base edge 4. Area of the base is $\frac{3}{2}(4^2)\sqrt{3}$ or $24\sqrt{3}$. So volume is

$\frac{1}{3}\left(24\sqrt{3}\right)\sqrt{13}$ or $8\sqrt{39}$.



15. **A**. $\pi 8^2 = 64\pi$

16. **C**. $18\pi = \frac{1}{3}\pi(r^2)(2r)$ gives r=3 and h=6.

17. **C**. The ratio of the areas is the square of the ratio of the radii, so the radii are in the ratio of 1:2 and the volumes' ratio is the cube of that, 1:8.

18. **B**. Draw the lines of symmetry for the square base and a plane of symmetry contains one of those lines and the vertex of the pyramid.

19. **E.** An infinite number. Consider one diameter, and the planes that contain it.

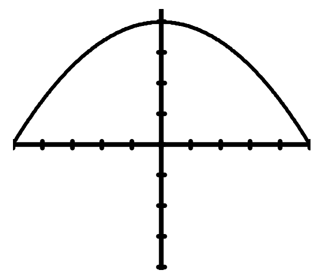
20. **B**. 5(36)+5(9)-9 is showing. 5(45)-9= 216. Total is 6(36)+6(9)=6(45) = 270. 216/270 reduces to 24/30=4/5

21. **B**. Area of the triangle cross section is $\frac{1}{2}(2r)h = 12$ so rh=12. If the radius and height are whole numbers then we have possibilities: 10 π is not one of them.

r	1	12	2	6	3	4
h	12	1	6	2	4	3
V	4π	48 π	8 π	24 π	12 π	16 π

22. **A.** This is a right triangle and the height is given by $\sqrt{2^2+4^2} = 2\sqrt{5}$. The area is 1/2 times 4($2\sqrt{5}$)= $4\sqrt{5}$.

23. **D**. The shape is a cone, and the slant height is the distance along the line from (0, 0) to (2, 1) which is $\sqrt{5}$.
LA is $\frac{1}{2}\sqrt{5}(2\pi)$ since the radius is 1 and circumference is 2π.



24. **D**. I am going to turn the parabola over and graph: Using vertex (0, 4) and x-intercept (5, 0) we get $(y-5) = a(x-0)^2$ and using (5, 0) we get a= -4/25.
So 1/(4a)=1/(16/25) and the distance from the vertex to the focus is 25/16.

25. **C**. Using 6 as the height, we use 4 as the circumference of the base, to get $2\pi r = 4$ and $r = \frac{2}{\pi}$. So volume is $\pi\left(\frac{2}{\pi}\right)\left(\frac{2}{\pi}\right)6 = \frac{24}{\pi}$

26. **B**. If 1/8 is filled then the small and similar cone at the top is 7/8 of the big cone. So the ratio of the volumes is 7/8 and the ratio of the heights is $\frac{\sqrt[3]{7}}{\sqrt[3]{8}}$. Using this ratio with a large cone height of 4 give small cone height $2\sqrt[3]{7}$

27. **C**. We have octagon faces and triangle faces. One octagon is 16 minus $\frac{1}{2}(\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}})$ times 4 or 16-1=15. There are 6 such faces. So that makes 90.

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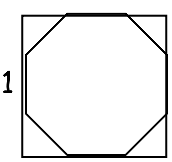


The triangles each have area $\frac{1}{4}\sqrt{3}$ and there are 8 of them. So surface area is $90+2\sqrt{3}$. A+B+C=90+2+3=95.

28. **D**. Set $\frac{4}{3}\pi r^3$ equal to 36π to get r=3 then use $4\pi r^2$ to get 36π.

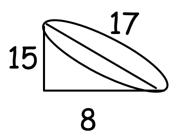
29.**C**. $\frac{364\pi}{3}$ =outer sphere minus the inner sphere. $\frac{4}{3}\pi(r+1)^3 - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left((r+1)^3 - r^3\right) = \frac{364\pi}{3}$ so

$91 = (r+1)^3 - r^3$ and using the formula for the difference of cubes gives

$(r+1-r)((r+1)^2 + r(r+1) + r^2) = 3r^2 + 3r = 90, r^2 + r = 30$ and $r^2 + r - 30 = 0$ so r=5 or -6,

choose r=5. Volume is therefore $\frac{500\pi}{3}$

30. **D**. In the tilted position we connect the low-water to the high water point to get a right triangle.



So the radius of the container is half of 8. The height will be the average of 20 and 35 since the water maintains its center – so we have height 55/2. Volume is then $\pi(16)\frac{55}{2}$ or 440π.