

### Answer Key

1. B	6. A	11. E	16. B	21. D	26. C
2. D	7. B	12. A	17. B	22. C	27. C
3. C	8. A	13. D	18. C	23. B	28. D
4. C	9. D	14. C	19. A	24. B	29. C
5. B	10. C	15. E	20. B	25. C	30. C

### Solutions

1. Expanding the left and right sides results in  $3x+3-x+3=2x-6-2x+2$ . Combining like terms yields  $2x+6=-4$ , which results in  $x=-5$ .
2. The formula for the sum of the roots of  $ax^3+bx^2+cx+d=0$  is  $-b/a$ . So the sum of the roots of  $f(x)$  is  $b/a$ . (Note the coefficient of  $x^2$  is  $-b$ .)
3. Note the portion in the denominator is equal to 5. So  $5=1+\sqrt{x}/5 \Rightarrow 20=\sqrt{x} \Rightarrow x=400$ .
4. Since  $f(f^{-1}(x))=x$ , we have  $\frac{1+f^{-1}(x)}{1-f^{-1}(x)}=x$ . Solving for  $f^{-1}(x)$  gives  $f^{-1}(x)=\frac{x-1}{x+1}$ .  
So  $f^{-1}(10)=\frac{9}{11}$ .
5. If  $2x+5 \geq 0$  and  $4x+7 \geq 0$ , then  $2x+5+4x+7 < 30 \Rightarrow x < 3$ . If  $2x+5 \leq 0$  and  $4x+7 \leq 0$ , then  $-2x-5-4x-7 < 30 \Rightarrow x > -7$ . Testing the other two cases where one is negative and the other positive result in  $x > -16$  and  $x < 14$ . The union of these four inequalities is  $-7 < x < 3$ .
6.  $(a+b+c)^2=(a^2+b^2+c^2)+2(ab+ac+bc)$ . So  $(a^2+b^2+c^2)=9/4-2(4/2)=-7/4$ . (Note: sum of squares is negative because roots are imaginary. Also, the formula for sum of roots taken 2 at a time can be found by expanding  $f(x)=(x-a)(x-b)(x-c)$ .)
7. An ellipse with center  $(h,k)$ , major axis of length  $2a$  and minor axis parallel to the  $x$ -axis of length  $2b$  is in the form  $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$ .
8. Completing the square and putting the equation into the standard form yields  $\frac{(x+2)^2}{4}+\frac{(y-3)^2}{9}=1$ .  
 $a^2=4, b^2=9$ . Area is  $ab\pi$ .
9.  $n+d=80$  and  $5n+10d=500$ . Solve the system to get 60 nickels and 20 dimes.
10. By definition,  $f(1)=0+f(0)=1$ .  $f(2)=1+f(1)=2$ .  $f(3)=2+f(2)=4$ .  $f(4)=3+f(3)=7$ .  
 $f(5)=4+f(4)=11$ .  $f(6)=5+f(5)=16$ .
11. Since you don't know whether  $x+2$  is negative you cannot cross multiply.  
 $\frac{x+3}{x-1}-\frac{x+5}{x+2} < 0$ ;  $\frac{(x+3)(x+2)-(x+5)(x-1)}{(x-1)(x+2)} < 0$ ;  $\frac{x+11}{(x-1)(x+2)} < 0$ ;  $x < -11 \cup -2 < x < 1$
12.  $x(x^2-4)-(x-2)+2(2-x)=x^3-7x+6=0$ . Product of roots is  $-6$ .
13. Divide both sides of both equations by  $xy$ , let  $a=1/x$  and  $b=1/y$  and solve the system.

14. Cross multiply to get  $8x + 7 = A(x - 1) + B(x + 2)$ . When  $x = 1$ ,  $B = 5$  and when  $x = -2$ ,  $A = 3$ .  
 $B - A = 2$ .
15. There are 3 possibilities. Exponent is 0, base is 1, or base is -1. If exponent is 0,  $x = 1$  or  $x = 2$ .  
 But note if  $x = 2$ , then the base is also 0.  $0^0$  is undefined. If the base is 1,  $x^2 - 6x + 8 = 1$ . this has  
 irrational solutions that sum to 6. If the base is -1,  $x^2 - 6x + 8 = -1 \Rightarrow x^2 - 6x + 9 = (x - 3)^2 = 0$ . If  
 the base is -1, the exponent must be even. At  $x = 3$ , the exponent is 2. So the sum of the solutions  
 is  $6 + 3 + 1 = 10$ .
16.  $f(g(3)) = f(5) = 11$ .  $g(f(3)) = g(13) = 15$ .  $11 - 15 = -4$ .
17.  $\log_3 x + \log_9 x^2 = \log_3 x + 2\log_9 x = 2\log_3 x = 4 \Rightarrow x = 9$ .
18. Move the 7 over to get  $(x - 6)(x + 2) < 0$ . Test intervals to find  $-2 < x < 6$ .
19. Product of the three roots is 27, so the geometric mean is  $\sqrt[3]{27} = 3$ .
20. Formula is  $A = P\left(1 + \frac{r}{n}\right)^{nt}$ , where  $r, n, t$  are rate, number of times a year, and time, respectively.  
 Every three months means 4 times a year.
21. Plug  $(x/2)$  into  $f(2x)$  to get  $f(x) = x^2/4 + x + 3$ . Product is 12.
22. Multiplying both sides by  $\ln x$  and rearranging gives  $(\ln x)^2 - 6(\ln x) + 7 = 0$ . Let  $u = \ln x$ .  
 So  $u^2 - 6u + 7 = 0$  and  $u_1 + u_2 = 6$ . Since  $u_1 = \ln x_1$  and  
 $u_2 = \ln x_2$ ,  $u_1 + u_2 = \ln x_1 + \ln x_2 = \ln x_1 x_2 = 6$ . So  $e^6 = x_1 x_2$ .
23. Discriminant is  $b^2 - 4ac$ . In this case,  $5^2 - 4 \cdot 3 = 13$ .
24. This factors into  $x^4 + 3x^2 - 4 = (x^2 + 4)(x^2 - 1) = (x^2 + 4)(x - 1)(x + 1)$ . 2 real roots.
25. If  $(x - 8) \geq 0$ , then  $11(x - 8) \leq 3x \Rightarrow x \leq 11$ . Else,  $11(8 - x) \leq 3x \Rightarrow x \geq 44/7$ . So  $44/7 \leq x \leq 11$ . 5  
 numbers fall between this, namely 7, 8, 9, 10, and 11.
26. When the expression is expanded completely, the sum of the coefficients will be the value of  
 expression when  $x = 1$ . So we can plug in  $x = 1$  to get  $2^4 = 16$ .
27. The top factors into  $(x - 1)(x + 3)(x - 2)$  and the bottom into  $(x - 2)(x + 2)(x + 1)$ .  $(x - 2)$  cancels  
 out, leaving two vertical asymptotes,  $x = -2$  and  $x = -1$  and a horizontal of  $y = 1$ , for a total of 3.
28. Using the Binomial Theorem, the term will be  $\binom{5}{3} (2x^2)^3 (-1/x)^2 = 10(8x^6)(x^{-2}) = 80x^4$ .
29. If we add the three equations together, we get  $2X + 2Y + 2Z = 108 = 2(X + Y + Z)$ .  
 So  $X + Y + Z = 54$ .
30.  $f(3) = 9 + 21 - 3 = 27$