2009 Mu School Bowl

1. Given \( f(x) = 2x^3 - 9x^2 + 12x + 1 \)

   \( A \) = the slope of the line tangent to \( f(x) \) at \( x = -3 \).

   \( B \) = the \( y \) – intercept of the line tangent to \( f(x) \) at \( x = 2 \)

   \( C \) = \( y \) – value of the point of inflection of the graph of \( f(x) \)

   \( D \) = the value of \( \int_{0}^{3} f(x)dx \).

   Find: \( \sqrt{A + B + C + D} \)

2. \( A = \text{det}\left( \begin{bmatrix} -2 & 5 & 1 \\ 3 & 7 & 0 \\ 4 & 0 & 5 \end{bmatrix} \right) \)

   \( B \) = the value of \( f(f(f(2))) \) if \( f(x) = 3x^2 - 2x - 1 \) when \( x \) is even and \( f(x) = x^2 - 5x + 1 \) when \( x \) is odd.

   \( C \) = the area enclosed by \( y = |x - 1| + |x + 2| \) and \( y = |x - 3| - |x + 2| \).

   \( D \) = \( \tan(\cos^{-1}\left(\frac{6}{7}\right)) \).

   Find: \( A - B + C + 13D^2 \).

3. \( A = \sum_{2}^{100} \frac{3}{x(x-1)} \)

   \( B \) = the sum of the real zeros of \( f(x) = x^4 - 3x^3 - 5x^2 + 23x + 66 \).

   \( C \) = the length of the graph \( f(x) = \ln x - \frac{1}{8}x^2 \) for \( 1 \leq x \leq 2 \).

   \( D \) = the derivative at the point \((-1, 2)\), given \( x^2 - xy^2 + y^3 = 13 \).

   Find: \( 100A - 4(B + C + D) \).
4. \( A = \) the sum of \( x \) and \( y \) in the solution to: \( 3(x + y)i - 3i + 2x = 5 + 7i \).

\[
B = \text{the value of } k \text{ in } \sqrt[\frac{a}{b}]{\frac{b}{a}} \sqrt[\frac{a}{b}]{\frac{b}{a}} = \left(\frac{a}{b}\right)^k \text{ for all } a, b \geq 0
\]

\( C = \) the sum of the positive value(s) of \( x \) if \( 4^{\log_2 x} + x^2 = 8 \)

\( D = \) length of the altitude to the longest side of a triangle with side lengths of 13, 37, and 40.

Find \( ABCD \)

5. Given: \( f(x) = 5x^5 - 4x^3 - 2x - 4 \).

\( A = \) the \( x \)-value of the positive inflection point.

\( B = \) the number of real zeros + the number of local extrema.

\( C = \) the area between \( f(x) \) and \( y = -x - 4 \) in the interval \([0, 1]\).

\[
D = \lim_{x \to \frac{\sqrt{6}}{5}} \frac{y''}{5x - \sqrt{6}}.
\]

Find: \( A^2 + BC - D \)

6. \( A = \) Area of the ellipse: \( 9x^2 + 4y^2 - 18x + 16y = 11 \)

\( B = \) the number of integer pairs that satisfy the conditions: \( y > |x - 1| + |x - 5| \) and \( y < 6 \).

\( C = \) is the eccentricity of the conic with an equation of: \( 4x^2 - y^2 + 24x - 10y - 15 = 0 \).

\( D = \) the length of the transverse axis of the hyperbola: \( x^2 - 4y^2 + 4x + 32y - 96 = 0 \)

Find: \( AB + CD \)
7. \( A = \) the number of positive integer pairs \((x, y)\) that are solutions to \(5x + 3y = 87\). 

\[ B = \text{the coefficient of the 6}^{\text{th}} \text{ term in the expansion of } (2x - y)^6. \]

\[ C = \text{the number of positive ordered pairs satisfying } x = \frac{6-x}{y^2-x}. \]

\[ D = \text{the simplified form of: } \left(3\sqrt[3]{75 - \sqrt{12}}\right)^{-2}. \]

Find: \( \frac{B}{ACD} \)

8. \( A = a + b + c, \) given: \( \sum_{2}^{2008} \log_2 \frac{n}{n+1} = a + b \log_2 c. \)

\[ B = \text{the sum of the solution(s) to: } \ln(3x - 1) + \ln(x + 3) = 2 \ln 5. \]

\[ C = \text{the value of } \log_n \frac{(a \sqrt{b})^2}{b^2c}, \text{ if } \log_n a = 2, \ \log_n b = 3, \ \log_n c = 5. \]

\[ D = k \text{ if } \log_y x + \log_y x = 6, \text{ then } x = y^k. \]

Find the sum of the digits in the simplified form of \( \frac{2A}{B} + CD. \)

9. \( A = f\left(\frac{1}{2}\right) \) if \( f'(x) = x\sqrt{1 - x^2} \) and \( f(0) = 3. \)

\[ B = \lim_{x \to \infty} \frac{\ln(x^2 + 1)}{\ln x}. \]

\[ C = \text{is the derivative of } y \text{ with respect to } x \text{ at } \left(\frac{3\pi}{4}, -2\right) \text{ of } y^2 \sin 2x = 2y. \]

\[ D = \text{the area } A \text{ of the region between the graph of } f(x) = 3x^2 + 4 \text{ and the } x - \text{axis} \]

\[ \text{in the interval } [-1, 1]. \]

Find: \( (A + C)(B + D) \)

10. \( A = \) the smaller value of \( x \) which satisfies \( \log_9 x + \frac{1}{\log_9 x} = \frac{5}{2}. \)

\[ B = \text{the smallest of 3 integers that form a geometric progression if their sum is 21 and the sum} \]
of their reciprocals is $\frac{7}{12}$.

$C$ = the area of rectangle $R_2$ if its diagonal is 15 and is similar to rectangle $R_1$ with one side 2 and area 12.

$D = a + b$ if $\sqrt{20} + \sqrt{384} = \sqrt{a} + \sqrt{b}$ and $a < b$.

Find: $\frac{C^2}{ABD}$