2009 Geometry Hustle

Solutions

1. 88^o $2x - 10 + 4x + 5x + 3 = 180 \Rightarrow 11x = 187 \Rightarrow x = 17$ $5x + 3 = 5(17) + 3 = 88$

2. 27 Number of diagonals = $\frac{1}{2}(n)(n-3) \Rightarrow$ $d = \frac{1}{2}(9)(9-3) = 27$

3. 10 Connecting the center to the end of the chord forms a right triangle
$6^2 + 8^2 = r^2$ which gives $r = 10$

4. $\frac{2\sqrt{30}}{3}$ The side opposite the 60^o angle is $\sqrt{3}$ times the short leg,
so $\sqrt{10}$ should be divided by $\sqrt{3}$. This gives $\frac{\sqrt{30}}{3}$ for the short leg and the hypotenuse is twice as long as the short leg and is $\frac{2\sqrt{30}}{3}$.

5. $4\pi^3$ If the diameter is 4π, then the radius is 2π. Area = $\pi r^2 = \pi(2\pi)^2 = 4\pi^3$ square units.

6. $k = \frac{25}{9}$ *or* $2\frac{7}{9}$ $2 \cdot \frac{25}{3} = 6 \cdot k \Rightarrow \frac{50}{3} = 6k \Rightarrow k = \frac{50}{18} = \frac{25}{9}$

7. $12\sqrt{2}$ The outside square has a side length of 6 meters ($24 \div 4$). The inscribed circle has a diameter of 6 meters, which is also the length of the diagonal of the inscribed square. Since the diagonal of a square makes a 45-45-90 triangle, the side of the square is the diagonal divided by $\sqrt{2}$. Simplifying $\frac{6}{\sqrt{2}}$ gives $\frac{6\sqrt{2}}{2} = 3\sqrt{2}$. Perimeter is 4 times the side, or $12\sqrt{2}$.

8. 324 cm^2 Area of a rhombus = $\frac{1}{2} d_1 d_2 = \frac{1}{2} \cdot 18 \cdot 36 = 324$

9. 24π in^2 By the Pythagorean Theorem the lateral height, l, is 5 in.
$SA = \pi rl + \pi r^2 = \pi \cdot 3 \cdot 5 + \pi \cdot 3^2 = 15\pi + 9\pi = 24\pi$ in^2

10. 4 or 4:1 Simplifying the area ratio: 184 : 736 = 1 : 4 which is the square of the ratio of the lengths of corresponding sides of the similar figures. The ratio of the sides is 1 : 2 and the ratio of the volumes is 1: 8. ½ ÷ ⅛ = 4

11. $\frac{5}{18}$ If each of the nine large squares on the target is divided into fourths, there are 36 small squares. 10 of them are shaded so the probability of hitting one of them is 10 out of 36.

12. 40 A dodecahedron has 12 surfaces or faces, a square pyramid has 8 edges, and an icosagon has twenty vertices. 12 + 8 + 20 = 40

13. $18\sqrt{5}$ Using the geometric mean property, BD = $\sqrt{20} = 2\sqrt{5}$.
By Pythagorean Theorem in ΔADB, AB = 6.
By Pythagorean Theorem in ΔBDC, BC = $3\sqrt{5}$.
Thus AB · BC = $6 \cdot 3\sqrt{5} = 18\sqrt{5}$.

14. 104° Draw in radius OA forming two congruent isosceles triangles. ∠BAC = 2 · 26 = 52°. An inscribed angle measures ½ of the intercepted arc, so $\widehat{BC} = 104°$.

15. 24 sides If the interior angles of a regular polygon measure 165°, then the exterior angles measure 15°. Since the sum of the exterior angles is 360°, divide 360 by 15 to get the number of sides, 24.

16. 40 in. The circular base of the cylinder has a radius of 12 and a diameter of 24 in. Slicing through the cylinder perpendicular to the base forms a right triangle whose hypotenuse will be the longest possible length of a stick that fits in the cylinder. Using the Pythagorean Theorem, that length is 40 in. ($24^2 + 32^2 = \text{diag}^2$)

17. $38\frac{2}{3}$ *or* $\frac{116}{3}$ x (3x + 2) = 65 => $3x^2 + 2x - 65 = 0$ => (3x – 13)(x + 5) = 0 so x = 13/3
Perimeter = 2 (x + 3x + 2) = 8x + 4 = 8 (13/3) + 4 = 116/3 or 38 2/3 units

18. Centroid

19. (-7, -2) (7, -2) reflected over the x-axis becomes (7, 2). If (7, 2) is rotated 180°, both the x and y coordinates change signs.

20. 35 The students who like Mexican or Thai or both is found by adding the Mexican and Thai and subtracting those who like both. This gives 65 students. (53 + 42 – 30) Subtracting 65 from the total of 100 students gives 35 who do not like either Mexican or Thai food.

21. $x^2 + (y - 3)^2 = 25$ The center of the circle is the midpoint of the ends of the diameter (0, 3). The radius is the distance from the center to one end of the diameter. Using the distance formula gives the radius as 5 units.

22. $\sqrt{29}$ miles Adding the north-south miles together and the east-west miles together shows that Tom ends up 5 miles east and 2 miles north of the KCC. Making KCC the point (0, 0) and Tom's position the point (5, 2) and using the distance formula tells us that Tom is $\sqrt{29}$ miles away.

23. 158° $\angle D = \frac{1}{2}(127 + 75) = 101°$. A quadrilateral's angles add to 360° so $\angle C$ is 360 – (85 + 79 + 101) or 95°. Angle C = ½ $\widehat{BAD}$ so the arc measures 190°. $\widehat{AD} = \widehat{BAD} - \widehat{BA} = 190 - 127 = 63°$. $\angle C + \widehat{AD} = 95° + 63° = 158°$

24. $\frac{3}{2}$ *or* 1.5 Sides of a rectangle are perpendicular, so the slopes of these two lines must give a product equal to -1. If $\frac{-3}{4} \cdot \frac{2}{k} = -1$, then $k = \frac{3}{2}$.

25. 62.5 sq. un. Working from left to right, the areas of the shapes formed are:
$\frac{1}{2} \cdot 5 \cdot 5 + 5 \cdot 5 + \frac{1}{2} \cdot 7.5 \cdot 5 + \frac{1}{2} \cdot 2.5 \cdot 5 = 12.5 + 25 + 18.75 + 6.25 = 62.5$