2009 MAθ HUSTLE Algebra Solutions

1. AB = $\begin{bmatrix} 6 & 0 \\ -12 & -9 \end{bmatrix}$

2. $\left(\frac{5}{6}, \frac{-13}{12}\right)$ x = $\frac{-(-5)}{2(3)} = \frac{5}{6}$ y = $3\left(\frac{5}{6}\right)^2 - 5\left(\frac{5}{6}\right) + 1 = \frac{25}{12} - \frac{50}{12} + \frac{12}{12} = \frac{-13}{12}$

3. (2, -1) $4(x^2 - 4x + 4) + 9(y^2 + 2y + 1) = 11 + 16 + 9 = 36$ $\frac{(x-2)^2}{9} + \frac{(y+1)^2}{4} = 1$

4. (6, 2) $\begin{cases} 3x - 5y = 8 \\ 2x + 3y = 18 \end{cases} \Rightarrow \begin{cases} 6x - 10y = 16 \\ -6x - 9y = -54 \end{cases} \Rightarrow -18y = -38 \Rightarrow y = 2$

3x -10 = 8 3x = 18 x = 6

5. $\sqrt{34}$ $(x^2 - 10x + 25) + (y^2 + 5y + 6.25) = 2.75 + 25 + 6.25 = 34$ r = $\sqrt{34}$

6. 5040 7·6·5·Catcher·4·3·2·1·Pitcher = 5040

7. 3780 $\frac{9!}{4!2!2!} = 3780$

8. 10 $\begin{vmatrix} 3 & 2 & -1 \\ -2 & 0 & 4 \\ 1 & -1 & -2 \end{vmatrix} = 0 + 8 - 2 - 0 + 12 - 8 = 10$

9. Multiplied by 8 Weight = $\frac{k \cdot width^2}{length}$ New wt. = $\frac{k \cdot (2\,width)^2}{\frac{1}{2} \cdot length} = \frac{4k \cdot width^2}{\frac{1}{2} \cdot length} = \frac{8\,k \cdot width^2}{length} = 8 \cdot orig.wt.$

10. (-3, 7) $\begin{cases} y = x^2 - 2 \\ y = 1 - 2x \end{cases} \Rightarrow x^2 - 2 = 1 - 2x \Rightarrow x^2 + 2x - 3 = 0 \Rightarrow (x + 3)(x - 1) = 0$

So, x = -3 or x = 1 giving (-3, 7) as the second quadrant point

11. 27 Each log term reduces to 3 and there are 9 terms -- so 9 · 3 = 27

12. 140 feet There are two infinite geometric series – one for the falling distances and one for the distances rebounded.
Falling, it travels $\frac{20}{1-\frac{3}{4}} = 80$ feet. Rising, it travels $\frac{15}{1-\frac{3}{4}} = 60$ feet. Total = 140 feet

13. Yes, $18\frac{2}{3}$ min. $\frac{x}{35} + \frac{x}{40} = 1 \Rightarrow 8x + 7x = 280 \Rightarrow x = 18\frac{2}{3}$ min. which is less than 20 min.

14. {-2, 3, 5} Synthetic division by possible rational roots leads to -2 as a root with a quotient of $x^2 - 8x + 15$ which factors into (x – 3)(x – 5) so 3 and 5 are the other roots.

15. x = 36 $x = 2\sqrt{x} + 24 \Rightarrow x - 2\sqrt{x} - 24 = 0 \Rightarrow \left(\sqrt{x} - 6\right)\left(\sqrt{x} + 4\right) = 0 \Rightarrow x = 36\ or\ 16$
Only x = 36 will check in the original solution.

16. x = -9.5 $4^{5x-2} = \left(\frac{1}{64}\right)^{7-x} \Rightarrow 4^{5x-2} = (4^{-3})^{7-x} \Rightarrow 5x - 2 = -21 + 3x \Rightarrow 2x = -19$
So x = -9.5

17. x = 8 $\log_3(x+1) + \log_3(x-5) = 3 \Rightarrow \log_3[(x+1)(x-5)] = 3 \Rightarrow x^2 - 4x - 5 = 3^3 = 27$

$x^2 - 4x - 32 = 0 \Rightarrow (x-8)(x+4) = 0 \Rightarrow$ x = 8 or - 4 but only 8 works since the domain of a logarithm must be positive.

18. (3, ½) $\begin{cases} \frac{3}{x} + \frac{4}{y} = 9 \\ \frac{-9}{x} + \frac{5}{y} = 7 \end{cases} \Rightarrow \begin{cases} \frac{9}{x} + \frac{12}{y} = 27 \\ \frac{-9}{x} + \frac{5}{y} = 7 \end{cases} \Rightarrow \frac{17}{y} = 34 \Rightarrow y = \frac{1}{2}$

Substituting gives $\frac{3}{x} + \frac{4}{\frac{1}{2}} = 9 \Rightarrow \frac{3}{x} + 8 = 9 \Rightarrow \frac{3}{x} = 1 \Rightarrow x = 3$

19. $y = \frac{-1}{6}x - \frac{5}{3}$ $9(x^2 + 2x + 1) - 4(y^2 + 3y + 2.25) = 36 + 9 - 9 = 36 \Rightarrow \frac{(x+1)^2}{4} - \frac{(y+1.5)^2}{9} = 1$

$4(x^2 - 4x + 4) + 5(y^2 + 4y + 4) = 4 + 16 + 20 = 40 \Rightarrow \frac{(x-2)^2}{10} + \frac{(y+2)^2}{8} = 1$

Centers are (-1, -1.5) and (2, -2) Slope = $\frac{-2+1.5}{2+1} = \frac{-.5}{3} = \frac{-1}{6}$

Using the point (2, -2) we get $y + 2 = \frac{-1}{6}(x - 2) \Rightarrow y = \frac{-1}{6}x - \frac{5}{3}$

20. 144 This is an arithmetic sequence with d = -5. $a_{10} = 9 + (10-1)(-5) = -36$

$a_{28} = 9 + (28-1)(-5) = -126$ $3a_{10} - 2a_{28} = 3(-36) - 2(-126) = -108 + 252 = 144$

21. a = -2, b = 7 Substituting p(-1) = 0 gives -1 –a – b + 6 = 0 which simplifies to a + b = 5

p(3) = 72 gives 27 – 9a + 3b + 6 = 72 which becomes 9a – 3b = -39

Solving those two equations together gives the solution (-2, 7).

22. (4, ½, -½) Eliminating z from the first two equations results in 7x – 8y = 24
Eliminating z from the first and third equations gives 6x – 10y = 19
Solving that system of two equations results in x = 4 and y = ½.
Substituting back into one of the original equations gives z = -½.

23. $\frac{4+\sqrt{41}}{2}$ *or* $2 + \frac{\sqrt{41}}{2}$ *sec.* $h(t) = -16t^2 + 64t + 100$ gives the height of the object at time t.
Making the height zero: $-16t^2 + 64t + 100 = 0$, dividing by -4, and using the quadratic formula gives $t = \frac{16 \pm \sqrt{256+400}}{8} = \frac{16 \pm \sqrt{656}}{8} = \frac{4 \pm \sqrt{41}}{2}$

24. $x \le -1$ or $x \ge 5$ Solve: $|x+4| \le 2x - 1$ OR $|x+4| \le -2x + 1$

$x+4 \le 2x-1$ AND $x+4 \ge -2x+1$ $x+4 \le -2x+1$ AND $x+4 \ge 2x - 1$

$x \ge 5$ $x \ge -1$ $x \le -1$ $x \le 5$

$x \ge 5$ $x \le -1$

25. 275 Expanding the sum = 30 + 29 + 28 + … + 20 = $\frac{11}{2}(30 + 20) = 275$