

2009 MAΘ Hustle Probability & Statistics Solutions

1. 588.8 $\{2, 3, 5, 8, 8, 8, 10, 15, 20, 21\}$ has a mean of $\frac{92}{10} = 9.2$; the median is 8; and the mode is 8. Thus the product is $9.2 \cdot 8 \cdot 8 = 588.8$
2. $\frac{5}{12}$ There are 36 outcomes when 2 dice are rolled. 15 of them have a sum > 7 .
3. 3780 $9! / (4! 2! 2!) = 3780$
4. 42 The first 12 Fibonacci numbers are 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, and 144. The lower quartile value is 2.5 and the upper quartile value is 44.5. $44.5 - 2.5 = 42$
5. 0.62 The mean is $62 / 10 = 6.2$ and the range is $12 - 2 = 10$. So $6.2 / 10 = 0.62$
6. $\frac{1}{6}$ Three primes can be rolled $\{2, 3, 5\}$. Getting a prime on the first roll has a probability of $\frac{1}{2}$ and getting a different prime the second time has a probability of $\frac{1}{3}$. $\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$
7. $\frac{2}{13}$ There are 6 red face cards and 4 kings but there are 2 red kings. $6 + 4 - 2 = 8$ successful draws out of 52 possible cards.
8. 1260 ${}_{10}C_2 \cdot {}_8C_2 = 45 \cdot 28 = 1260$
9. 16.8 St. dev. = $\sqrt{var.} = 1.5$ lbs. Thus $13.8 + 2(1.5) = 16.8$ lbs.
10. $\frac{11}{32}$ The row of Pascal's Triangle with 6 as the second term is 1 6 15 20 15 6 1. These numbers represent the number of times heads occur 6, 5, 4, 3, 2, 1, and 0 times, so getting at least 4 heads occurs $1 + 6 + 15 = 22$ times out of the total of 64 times.
11. $\frac{1}{3}$ There are 90 females and 30 of them love pizza.
12. $\frac{216}{625}$ $P(2W \text{ and } 2L) = \frac{4!}{2!2!} \cdot \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} = 6 \cdot \frac{36}{625} = \frac{216}{625}$
13. 51 85% of 60 = 51 correct answers
14. $\frac{4}{15}$ $18 + 14 - 10 = 22$ who like one or both candies. Thus, $30 - 22 = 8$ do not like either.
15. 92 $76 + 83 + 91 + 72 + 82 + 2x = 7(84) \rightarrow 2x + 404 = 588 \rightarrow x = 92$
16. \$1.00 Exp. Value = $\$11 \cdot \frac{2}{7} - \$3 \cdot \frac{5}{7} = \$1.00$
17. 83.85% 34% lies within one standard deviation below the mean and $(34 + 13.5 + 2.35)\%$ lies within three standard deviations above the mean, so $34\% + 49.85\% = 83.85\%$

18. 2.2
$$z = \frac{\bar{x} - \mu}{s.d./\sqrt{n}} = \frac{14.55 - 14}{4.25/\sqrt{289}} = \frac{0.55}{4.25/17} = \frac{0.55(17)}{4.25} = 2.2$$

19. $\frac{81}{196}$
$$P(x \leq 4.5) = \frac{\text{area left of } x=4.5}{\text{total area of triangle}} = \frac{0.5 \cdot 4.5 \cdot \frac{9}{14}}{0.5 \cdot 7 \cdot 1} = \frac{81/56}{7/2} = \frac{81}{196}$$

20. $\frac{\sqrt{3}}{4}$
$$\text{s.d.} = \sqrt{p(1-p)} = \sqrt{0.25(1-0.25)} = \sqrt{\frac{1}{4} \cdot \frac{3}{4}} = \frac{\sqrt{3}}{4}$$

21. 80
$$(120, 15) \times 2 = (240, 30) - 30 = (210, 30) \div 3 = (70, 10) \quad 70 + 10 = 80$$

22. $\frac{7}{32}$
$$P(5H, 3T) = {}_8 C_3 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^3 = 56 \cdot \frac{1}{256} = \frac{7}{32}$$

23. $\frac{225}{512}$
$$\text{Titans need 2W and 1L to be 12-4. } P(2w, 1L) = \frac{3!}{2!1!} \left(\frac{5}{8} \cdot \frac{5}{8} \cdot \frac{3}{8}\right) = \frac{225}{512}$$

24. 3
$$y = 4x - 3 \Rightarrow 9 = 4x - 3 \Rightarrow 12 = 4x \Rightarrow x = 3$$

25. 0
$$\text{Values lie on } [-1, 1] \text{ so the average is 0.}$$