Directions: E is none of these is correct.

1. If one endpoint of a segment is (-2, 5) and its midpoint is (4, 9), find the length of the segment.
   a) 2\sqrt{5}  
   b) 4\sqrt{13}  
   c) 4\sqrt{5}  
   d) 8\sqrt{2}

2. Find the coordinates of K.
   a) \left(\frac{3}{2}, \frac{5}{2}\right)  
   b) \left(\frac{a+7}{2}, \frac{b+a+3}{2}\right)  
   c) (4, 1)  
   d) \left(\frac{5}{2}, \frac{3}{2}\right)

3. Find the equation of FH.
   a) 3x - 5y = 7  
   b) 5x + 3y = -8  
   c) 5x + 3y = 17  
   d) 3x + 5y = 32

4. Find the sum of a + b.
   a) -2.75  
   b) -5  
   c) -1  
   d) 3

5. What is the area of the kite.
   a) 25.25  
   b) 32  
   c) 34  
   d) 40.5

6. The regions defined by \(|x + 2y| \leq 2\) and \(|-4x - 2y| \leq 8\) intersect to form what shape?
   a) Rhombus  
   b) Rectangle  
   c) Kite  
   d) Square

7. Find the area enclosed by the graph of \(r^2 = \frac{36}{1+8 \sin^2 \theta}\).
   a) 6\pi  
   b) 12\pi  
   c) 18\pi  
   d) 24\pi

8. The points (3, 7), (6, 2), and (2, k) are the vertices of a triangle. For how many real values of k is the triangle a right triangle?
   a) 1  
   b) 2  
   c) 3  
   d) 4

9. Which of the following is a polar representation of the Cartesian coordinate relation \(y = x^2\)?
   a) \(r = \theta^2\)  
   b) \(r = \sec \theta \tan \theta\)  
   c) \(r = \cos^2 \theta\)  
   d) \(r = \cos \theta \cot \theta\)
10. Which of the following represents the set of points twice the distance from (-1, 2) as from (4, 6).

a) \(3x^2 + 3y^2 - 34x - 44y + 203 = 0\)  
b) \(3x^2 + 3y^2 + 16x - 4y - 26 = 0\)  
c) \(3x^2 + 3y^2 + 16x - 4y - 16 = 0\)  
d) \(3x^2 + 3y^2 - 16x + 4y + 56 = 0\)

11. Find the shortest distance between the point (1, 0, 0) and the line defined by \(x = t + 1, \ y = 2t - 1,\) and \(z = 3t - 4.\)

a) 1  
b) 1.5  
c) \(\sqrt{3}\)  
d) \(\sqrt{6}\)

**The points A(-1, 2), B(3, 4) and C(5, 2) lie on circle O. Use this information to answer the questions #12 - #14.**

12. What is the area of the circle?

a) \(5\pi\)  
b) \(10\pi\)  
c) \(12\pi\)  
d) \(16\pi\)

13. Find the distance the chord with endpoints (-1, 2) and (5, 2) is from the center of the circle.

a) 1  
b) 1.5  
c) 2.5  
d) 3

14. Find the area of the shaded region. (Given \(\triangle ABC\))

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<td>a) (5\pi - 6)</td>
<td>b) (10\pi - 7)</td>
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<tr>
<td>c) (12\pi - 6)</td>
<td>d) (12\pi - 10)</td>
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15. The points (3, 0, 1), (2, -2, 1), and (-2, -4, 2) define a plane. Which of the following points is also on the plane?

a) (5, 1, 1)  
b) (1, 3, 2)  
c) (2, 4, 2)  
d) (-1, -3, 3)

16. A football’s path can be described by the parametric equations \(x = 4t\) and \(y = 40t - 16t^2.\) Find a particular equation in \(x\) and \(y\) that describes the path of the ball.

a) \(y = 2x(5 - 4x)\)  
b) \(y = 2x(5 - 8x)\)  
c) \(y = x(10 - x)\)  
d) \(y = x(5 - 8x)\)

17. What is the measure of the acute angle (in radians) of the intersection of the lines \(y = 3x + 3\) and \(y = 1 - x?\)
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18. What is the eccentricity of the conic section defined by the set of points \((x, y)\) such that the distance between \((x, y)\) and \((-2, 0)\) is twice the distance between \((x, y)\) and \((1, 0)\)?

a) 0  


b) \(\frac{\pi}{4}\)  

c) \(\frac{\pi}{2}\)  

d) \(\frac{3\pi}{4} - \arctan(3)\)

19. Two distinct lines with slopes \(m_1\) and \(m_2\) with \(m_1 < m_2\) pass through the point \((-6, 7)\) such that distance between the lines and the origin is 2. What is the value of \(8m_1 + 4m_2\)?

a) 0  


b) -12  

c) -14  

d) -18

20. Which of the following is closest to the value of the area contained by the graphs of both \((x - 1)^2 + (y - 2)^2 = 16\) and \((x - 3)^2 + (y - 2)^2 = 16\)? Use \(\tan^{-1}(\sqrt{15}) \approx 75^\circ\).

a) \(\frac{40\pi}{3} - 4\sqrt{15}\)  


b) \(\frac{4\pi}{3} - 2\sqrt{15}\)  

c) \(18\pi - 4\sqrt{15}\)  

d) \(20\pi - 4\sqrt{15}\)

21. Given the function \(f(x, y) = \frac{1}{2x^2 + 3y}\), what is the maximum value of \(f(x, y)\) if the chosen coordinate \((x, y)\) must lie on the line segment with endpoints \((3, 0)\) and \((0, 1)\)?

a) \(\frac{1}{5}\)  


b) \(\frac{8}{23}\)  

c) \(\frac{1}{3}\)  

d) \(\frac{2}{5}\)

22. The length of the latus rectum of a parabola is 4 and the focus is located at \((2, 3)\). Given that the parabola has standard form \(y = ax^2 + bx + c\) where \(a < 0\), find the vertex of the parabola.

a) \((2, 1)\)  


b) \((2, 2)\)  

c) \((2, 4)\)  

d) \((2, 5)\)

23. Consider the point \(P(1, 0)\) on the ellipse given by the equation \(4x^2 + y^2 = 4\). There are two points \((a, b)\) and \((c, d)\) on the ellipse whose distance from \(P\) is a maximum. What is the value of \(abcd\)?

a) \(-\frac{32}{81}\)  


b) \(-\frac{8}{81}\)  

c) \(-\frac{32}{9}\)  

d) \(-\frac{8}{9}\)

24. Find the radius of the sphere with equation \(x^2 + y^2 + z^2 - 2x - 6y + 8z - 38 = 0\)

a) 4  


b) 8  

c) 16  

d) 64

25. Find the distance between the polar graph \(r = 2\sqrt{3}\cos(\theta)\) and the polar coordinate \((4, 15\phi)\).

a) \(4 - \sqrt{3}\)  


b) \(3\sqrt{3} - \sqrt{3}\)  

c) \(\sqrt{3} - \sqrt{3}\)  

d) \(2\sqrt{3}\)
2009 Open Analytical Geometry

26. Find the distance from the center to a focus of the hyperbola $3x^2 - 12x - 8y^2 + 8y - 38 = 0$.

   a) $\sqrt{11}$   b) 4   c) $\sqrt{22}$   d) 5

27. Consider parabola P which has directrix $y = 0$ and contains the point (3, 4). Find the distance between the focus of P and the point (3, 4).

   a) 3   b) 4   c) 5   d) 6

**Using the following information to answer questions #28 - #30: Triangle ABC is formed using $A(-5, -2)$, $B(3, 5)$ and $C(6, -3)$ as its vertices.**

28. What is the equation of the altitude to side $AC$?

   a) $x + 11y = 58$   b) $x - 5y = 10$   c) $11x - y = 52$   d) $11x - y = 28$

29. What is the length of the median to side $AB$?

   a) $\frac{\sqrt{277}}{2}$   b)   c)   d)

30. The point of intersection of the altitude to $AC$ and the median to $AB$ is $\left(\frac{a}{b}, \frac{c}{b}\right)$, find $a + b + c$.

   a) 1063   b) 687   c) 447   d) -1455

Tie-Breakers:

1. Isosceles triangle ABC is defined by the points $A(0, 0)$, $B(2, 5)$, and $C(-2, 5)$. The coordinates $D(0, y_1)$ and $E(0, y_2)$ on the y-axis have the property that $DB + DC = AD$ and $EB + EC = AE$. What is $y_1, y_2$? Note: The notation $DB$ denotes the distance from D to B.

2. A pyramid has a square base with edge length 2 and height 4. What is the cosine of the dihedral angle formed by two of the triangular sides?

3. Find the equation of the ellipse with vertices $(4, -2)$ and $(4, 8)$ and minor axis 6.