2009 Discrete Math (Open)

Solutions

Solutions:
1) **D** with OR, 1 makes the answer always 1. 1 is a dominator.
2) **C** ln x, x^2, e^x, x! II, I, IV, III
3) **B** since 5 shares a factor with 10, there is no reciprocal for 5.
4) **B** $\sum_{j=1}^{n}(j+1) = \frac{i(i+1)}{2}+i = \frac{1}{2}i^2+\frac{3}{2}i \quad \frac{1}{2}\sum_{i=1}^{4}i^2+\frac{3}{2}\sum_{i=1}^{4}i = \frac{1}{2}\frac{n(n+1)(2n+1)}{6}+\frac{3}{2}\frac{n(n+1)}{2}$

$= \frac{1}{2}\frac{4(4+1)(2\cdot 4+1)}{6}+\frac{3}{2}\frac{4(4+1)}{2} = \frac{4\cdot 5\cdot 9}{2\cdot 6}+\frac{3\cdot 4\cdot 5}{4} = 30$

5) **A** $\frac{11!}{4!4!2!} = 34650$ sum is 18
6) **C** 2016 = $2^5\cdot 3^2\cdot 7$ so 6*3*2 = 36
7) **D** The reciprocal of the identity is still the same. 8 1's on the main diagonal answer 8
8) **D** The reciprocal is 4. 4 * 2 = 8 which is 1 mod(7)
9) **B** The circuit adds to bits on lines A and B together with a Carry bit on line C. The answer ends up at P and the answer's carry in D.
10) **C** the truth table for II and III are identical
11) **A** a bijection can be defined between each pair of sets. All sets have same cardinality $\aleph_o$ (aleph-null)
12) **E**
13) **E** y is a function. The vertical line test only applies to Cartesian graphs.
14) **B** A relation is reflexive if ARA is true. A R A → A^2+A^2=1 not always true
A relation is symmetric if A R B → B R A A R B → A^2 + B^2=1 B R A → B^2 + A^2=1 this is true because addition is commutative.
A relation is transitive if A R B and B R C → A R C false A^2 + B^2 = 1 and B^2 + C^2 = 1 implies A^2 + C^2 = 2 – 2 B^2 false

15) **C** Convert circuit to Boolean and simplify.
$\overline{(A\oplus \overline{A}B)(B+C)}\ \overline{C}$ **= 1** for an AND to be true both sides must be true so
C =0 and $\overline{(A\oplus \overline{A}B)(B+C)}$ = 1 reverse the not
$(A\oplus \overline{A}B)(B+0) = 0$ substitute the C=0
$(A\oplus \overline{A}B)(B) = 0$ try both cases for B
If B=0 then A = * (it doesn't matter what A is)
If B = 1 then $A\oplus \overline{A}*1 = 0$. This implies $A\oplus \overline{A} = 0$. contradiction! so B can't be 1, it must be 0
So the triplets are (*, 0 ,0).

16) **D** To find the paths of lengths two, square the adjacency matrix. All entries now represent the paths of length 2. matrix squared is $\begin{matrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 \end{matrix}$ sum is 9. (note: the 2 represents the 2-paths from c to c. These two paths are created from the original as follows: c->b->c and c->d->c)

17) **A** 12 paths We must make 4 new adjacency matrices, square each and sum the entries.

$$\text{A)}\ \begin{matrix}1&1&1&0\\0&0&1&0\\0&1&0&1\\0&0&1&0\end{matrix} -> \begin{matrix}1&2&2&1\\0&1&0&1\\0&0&2&0\\0&1&0&1\end{matrix} = 12 \quad \text{B)}\ \begin{matrix}1&1&1&0\\0&0&1&0\\1&1&0&1\\0&0&1&0\end{matrix} = \begin{matrix}2&2&2&1\\1&1&0&1\\1&1&3&0\\1&1&0&1\end{matrix} -> 18 \quad \text{C)}$$

$$\begin{matrix}1&1&0&0\\0&0&1&0\\1&1&0&1\\0&0&1&0\end{matrix} = \begin{matrix}1&1&1&0\\1&1&0&1\\1&1&2&0\\1&1&0&1\end{matrix} -> 13 \quad \text{D)}\ \begin{matrix}1&1&1&0\\0&0&1&0\\0&1&1&1\\0&0&1&0\end{matrix} = \begin{matrix}1&2&3&1\\0&1&1&1\\0&1&3&1\\0&1&1&1\end{matrix} -> 18$$

18) **D**

$B86_{16} = \dfrac{B}{16} + \dfrac{8}{256} + \dfrac{6}{4096} = \dfrac{1475}{2048}$ **(D)**

19) **A 47**
f(12,3) = f(12/2, 3-1) + 3*12 = f(6,2) +36 = 11 + 36 = 47
f(6,2) = f(2-2,6+1)+6 = f(0,7)+6 = 5+6 = 11
f(0,7) = f(0/2, 7-1) + 3*0 = f(0,6) = 5
f(0,6) = f(6-2, 0+1)+ 0 = f(4,1) = 5
f(4,1) = f(4/2, 1-1) + 3*4 = f(2,0)+12 = -7 + 12 = 5
f(2,0) = f(0-2, 2+1) +2 = f(-2,3) +2 = -9 + 2 = -7
f(-2,3) = f(-2/2, 3-1) + 3(-2) = f(-1,2) –6 = -3-6 = -9
f(-1,2) = $(-1)^2 - 2^2$ = -3

20) **D**
^ / * ! 12 18 # 2 3 ^ ! 6 8 2 ! 9 15
^ / * 6 # 2 3 ^ 2 2 3
^ / * 6 6 ^ 2 2 3
^ / * 6 6 4 3
^ / 36 4 3
^ 9 3
729

21) **D** Matrix D is almost the identity matrix. It multiplies the 2^{nd} row by 2.
22) **E** All statements are true. The distributive law holds for Addition over Multiplication in Boolean.

23) **A** Mutually exclusive events are never independent. Since A and B are mutually exclusive, we know that if A happens, B can't. There is a relationship between A and B, so they are not independent. By definition of independent: A and B are independent if $P(A\cap B) = P(A)\cdot P(B)$ since P(A)>0 and P(B)>0 the product must be >0 but mutally exclusive says that $P(A\cap B) = 0$

24) **C** $P\left(R_1 \mid R_2\right)=\dfrac{P\left(R_1 \text{ and } R_2\right)}{P\left(R_2\right)}=\dfrac{\frac{2}{3}\cdot\frac{4}{5}}{\frac{2}{3}\cdot\frac{4}{5}+\frac{1}{3}\cdot\frac{2}{4}}=\dfrac{16}{21}$

25) **A** This is the probability of getting a heart when 4 are already removed from the deck.

26) **C** No mention is made about what the "first" card is, so it doesn't affect the probability. The probability of drawing an Ace is 1/13

27) **C** Since parenthesis COMPLETELY define digits, we have to use the new digits with standard base 4 columns.

$\left(\frac{1}{2}\right)(-1)(5)(3)$ thus ½ is worth ½ of 64 = 32. -1 = -16. 5*4 = 20 and 3. Summing = 39

28. **D** This is the famous Monty Hall Problem. The probability of winning with the door changing strategy is $\frac{2}{3}$. This problem is very contra-intuitive, so don't accept a dispute. If you don't believe me, go online.

Solution: The strategy is to try to pick a goat on the first try. The host will then show you the other goat. Switching (in this situation) is a guaranteed win so no change in probability. The probability of picking a goat on the first try is 2/3.

29. **B** This is from the book "Proofs Without Words" for the sum of the cubes. If you consider one of the "pyramid" shapes to represent the sum of squares (i.e. each level is a "square") you can see that they fit together to form a rectangular solid with the dimensions shown. This will work for all values of "n" even though the diagram only shows for n=4. Note: problem asks for which answer "best demonstrates"

30. **D** Universe: kittens a = green-eyed; b=loving fish; c=tailed; d=teachable; e=whiskered; h=willing to play with a gorilla. Translations

1) $\overline{d} \rightarrow \overline{b} \Leftrightarrow b \rightarrow d$
2) $h \rightarrow c$
3) $e \rightarrow b$
4) $a \rightarrow \overline{d} \Leftrightarrow d \rightarrow \overline{a}$
5) $c \rightarrow e$ so $h \rightarrow c \rightarrow e \rightarrow b \rightarrow d \rightarrow \overline{a}$ No kitten with green eyes will play with a gorilla