

Alpha Ciphering 2009

o. Find the number of permutations in the word Mississippi.

1. One of the roots of $x^3 - 2x^2 - 5x + 6 = 0$ is 1. Find the other 2 roots.

2. A square is inscribed in another square such that the vertices of the inscribed square divides the sides of the outside square in a ratio of 3:1. If the difference in the area of the 2 squares is 16 sq. units, find the area of the larger square.

3. Simplify: $\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{2009} - \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{2009}$

4. Find the sum of all real x such that $\sqrt{\frac{x+4}{x-1}} + \sqrt{\frac{x-1}{x+4}} = \frac{5}{2}$.

5. $\log x = 2; \log y = -3; \log z = 5$. Find the numerical equivalent of $\log \frac{x^2 \sqrt{xyz}}{y^3 \sqrt[3]{z}}$.

6. Solve: $\tan 2\theta + 2\sin \theta = 0$ where $0 \leq \theta < 2\pi$.

7. Solve for X ; $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} X = 2 \begin{bmatrix} -1 & 4 & 5 \\ 2 & 3 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & 0 \\ 3 & 2 \end{bmatrix}$

8. Solve the system:
$$\begin{cases} 5xy + 13y^2 + 36 = 0 \\ xy + 7y^2 = 6 \end{cases}$$

9. When a certain polynomial is divided by $x - 2$, the remainder is 2. When the polynomial is divided by $x + 2$, the remainder is -2. What is the remainder when the polynomial is divided by $x^2 - 4$?

10. Let $f(0) = -2, f(1) = 3$ and $f(n + 1) = 2f(n) - f(n - 1)$ for $n > 1$. Find $f(2009)$