1. E. 2011 Find the number of ordered pairs of real numbers (a, b) such that $(a+bi)^{2009} = a-bi$ SOLUTION $z = a+bi$ and $|z|^{2009} = |z|$. Gives $z = 0$ or $|z| = 1$. Multiply original equation by z to get $z^{2010} = |z| = 1$, which has 2010 solutions. Count 0 as a solution also.

2. E. 25 A ball is thrust up vertically from the ground into the air and hits the ground 2.5 seconds later. What is the maximum height of the ball in feet?. Assume that air resistance is negligible. **SOLUTION** $y = v_0t - 16t^2$, Since $y(2.5) = 0$, then $v_0 = 8$ and $y(5/4) = 25$

3. D. 5/2 If the geometric series $a + ar + ar^2 + ...$ has a sum of 7, and the terms involving odd powers of r have a sum of 3. What is $a + r$? **SOLUTION** $\frac{a}{1-r} = 7$ and $\frac{ar}{1-r^2} = 3$ and then $a = 7/4, b = 3/4$

4. E. 7/2 If $y' = |1 - x|$ and $y(0) = 1$, what is $y(3)$? **SOLUTION** $y(1) - y(0) = \int_0^1 (1-x)dx$ and $y(3) - y(1) = \int_1^3 (x-1)dx$

5. B. 0 If the graph of $y = x^3 + ax^2 + bx - 4$ has a point of inflection at (1, -6), what is the value of b? **SOLUTION** $y'' = 6x + 2a$ and inflection at $x - 1$ gives $a = -3$. Use (1, -6) in y to get b.

6. C. 4851 How many different points in 3-dimensional space have 3 positive integral coordinates whose sum is 100? **SOLUTION** Count the choices. With 1 in the first position, there are 98 choices in the second position , with 2 in the first position, there 97 choices in the second position, so $98 + 97 + ... + 1 = 98 \times 99/2$

7. D. $\frac{y-2x}{2y-x}$ Given that $x^2 + y^2 = xy$, find $\frac{dy}{dx}$. **SOLUTION** $2x + 2xyy' = y + xy'$

8. B . 4 Find this limit. $\lim_{x\to(\pi/2)^-} \frac{4\tan x}{1+\sec x}$ **SOLUTION** L'Hopital's rule gives limit of $4secx/tanx) = 4/sinx$

9. C. 60 I drove at a constant speed for 3 hours and traveled x km. If I had driven each kilometer one minute faster, I would have driven 30 km. further in the 3 hours. What is the value of x? **SOLUTION** original speed is $x/180$, new speed $\frac{x}{180-x}$, solve $x + 30 = 180(\frac{x}{180-x})$

10. C. 1, -4/3 Find all k so that (1, 2k), (3k, 4), and (5, 6k) cannot determine 3 points on a circle. **SOLUTION** Check when all three points lie on a line.

$$\frac{4-2k}{3k-1} = \frac{6k-4}{5-3k}$$

11. A. $\frac{3}{8\pi}$ Water is poured into a conical cup at the rate of 2/3 cubic inches per second. If the cup is 6 inches tall and the top of the cup has a radius of 2 inches, how fast does the water level rise when the water is 4 inches deep? (in inches per second), **SOLUTION** $w = \pi r^2 h/3$. Similar triangles give $h = 3r$ and $w = \pi r^3$ and $w' = \pi 3r^2 r'$ and $2/3 = \pi 3(4/3)^2 r'$ and $r' = \frac{1}{8\pi}$ asked for h'
12. D. 17 What is the base of the system in which 121 represents the same number as the decimal number 324? **SOLUTION** $b^2 + 2b + 1 = 324$ and $(b - 17)(b + 19) = 0$
13. A: $-3 < x < 1$ On what open interval is $f(x) = (x^2 - 3)e^x$ decreasing? **SOLUTION** $f' = (x^2 - 3)e^x + 2xe^x$ and $(x + 3)(x - 1) \leq 0$
14. B. 1 Calculate $\lim_{n\to\infty}(1 + \frac{1}{n^2})^{\sqrt{n}}$. **SOLUTION** Take ln of the expression to $\lim \frac{\ln(1+n^2)}{\frac{1}{\sqrt{n}}}$ use L'Hopital's rule and $\lim \frac{1}{(n^2+1)n^{1/2}}$
15. C. $2e^2$ Evaluate $\int_1^4 e^{\sqrt{x}} dx$. **SOLUTION** Let $u = \sqrt{x}$ and get $\int_1^2 2ue^u du$ and integrate by parts
16. C. $\frac{152\pi}{3}$ The region bounded by the curve $f(x) = 2\sqrt{x}$, the x-axis, and the lines $x = 4$ and $x = 9$, is revolved about the x-axis. Find the volume of the resulting solid. **SOLUTION** $\int_4^9 2\pi 4x dx = 30\pi$
17. C: $3(\sqrt{37} - 1)/2$ $A = \sqrt{6 + \sqrt{6 + \sqrt{6 + ...}}}$ and $B = \sqrt{9 - \sqrt{9 - \sqrt{9 - ....}}}$ Find AB. **SOLUTION** use $A^2 = 6 + A$ and $B^2 = 9 - B$ and get $A = 3, B = (\sqrt{37} - 1)/2$
18. A: 12 A sphere is inscribed in a cube. The cube has a surface area of 36 square meters. A second cube is then inscribed in the sphere. What is the surface area of the inner cube in square meters? **SOLUTION** Use s as side of cube, $6s^2 = 36$, $s = \sqrt{6}$, radius of sphere $\sqrt{6}/2$. distance from the one corner of the inscribed cube to opposite corner is 2 (radius of the sphere). Labeling one corner of the inscribed cube as (0,0, 0) and the opposite corner (x, x, x), gives the length of that diagonal as $x\sqrt{3}$. Then $x\sqrt{3} = \sqrt{6}$ and $x = \sqrt{2}$ and surface area is 12.
19. B. $\frac{28}{27}$ Find the value of $a + b + c + d$ so that the graph of $y = ax^3 + bx^2 + cx + d$ has a local minimum at (0,0) and a local maximum at (3,4). **SOLUTION** $y' = 3ax^2 + 2bs + c$ Then $y'(0) = 0$ gives $c = 0$. $(0, 0)$ on graph gives $d = 0$, $y'(3) = 27a + 6b = 0$ and $y(3) = 4 = 27a + 9$. $a = -8/27$ $b = 4/3$

20. C. $3\sqrt{210}$ $f(x) = \int_1^{3x} \sqrt{t^2 - t}\, dt$ for all $x > 1$. Find $f'(5)$. **SOLUTION** $f' = 3\sqrt{9x^2 - 3x}$

21. E. -2 Find $\lim_{x\to-\infty}(\sqrt{x^2+4x+7}) + x)$. **SOLUTION** Multiplyng by the conjugate

$$\lim_{x\to-\infty} \frac{4x+7}{\sqrt{x^2+4x+7}) - x}.$$

Divide top and bottom by x and recall that $x = -\sqrt{x^2}$ for taking inside that denominator square root and get

$$\lim \frac{4 + 7/x}{-\sqrt{1 + \frac{4}{x} + \frac{7}{x^2}} - 1}$$

22. E. 3 An open box with a rectangular base is to be constructed from a rectangular piece of cardboard 16 inches wide and 21 inches long by cutting out a square from each corner and then bending up the resulting sides. Find the size of the corner square that will produce a box having the largest possible volume. **SOLUTION** $V = x(16-2x)(21-2x)$ with x being the length of cutout square. $V' = (28-3x)(3-x)$ and check that max occurs at 3.

23. A. 4/9 If an arc of 45^o on a circle A has the same length as an arc of 30^o on circle B, then the ratio of the area of circle A to the area of circle B is : **SOLUTION** (circumferenceA)(45/360) = (circumferenceB)(30/360) , and radiusA /radius B = 2/3 and then get area ratio.

24. B. ln(45/4) Evaluate $\displaystyle\int_2^3 \frac{4x^2+13x-9}{x^3+2x^2-3x} dx$. **SOLUTION** Integrand becomes

$$\frac{3}{x} + \frac{-1}{x+3} + \frac{2}{x-1}$$

after integration, you get the following expression evaluated at 3 minus that expression evaluated at 2.

$$\ln \frac{x^3(x-1)^2}{x+3}$$

25. D. $\frac{e^2}{2}$ Assign an area to the region that lies under the graph of $y = e^{2x}$, over the x-axis and to the left of $x = 1$. SOLUTION $\int_{-\infty}^{1} e^{2x} dx$

26. C. 4/3 A particle travels along the x-axis with velocity given by $v(t) = 4 - 3t - t^2$. If the particle is at position 2 on the x-axis at $t = 0$, then where is the particle at $t = 2$? **SOLUTION** $s = 4t - t^3/3 - 3t^2/2 + 2$

27. E. 10 Calculate $\sum_{n=1}^{\infty}[\frac{7}{n(n+1)} + \frac{2}{3^{n-1}}]$ **SOLUTION** Rewrite first sum as $\frac{7}{n} - \frac{7}{n+1}$ and use cancelation of terms to get a sum of 7 and the second sum is geometric series with sum of 3.

28. E. $5\sqrt{3}/3$ Find the slope of the polar graph $r = \sin(2\theta)$, when $\theta = \pi/6$. **SOLUTION** $x = r\cos\theta = \sin 2\theta \cos\theta$ and $y = \sin 2\theta \sin\theta$ and

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

29. C. $\sqrt{3}/2$ Find the cosine of the acute angle between the lines tangent to the curves $f(x) = -\ln(\cos(x))$ and $g(x) = \ln(\sin(x))$at $\pi/3$. **SOLUTION** $f' = \tan x$ and $g' = \cot x$ angle of tangent line to f is Use $\cos\theta = \cos(\theta_2 - \theta_1) = \cos\theta_2\cos\theta_1 + \sin\theta_2\sin\theta_1$ with $\cos\theta_2 = 1/2$ and $\cos\theta_1 = \sqrt{3}/2$

30. D. 10 Solve for $y(3)$, such that $y' = \frac{2xy}{1+x^2}$ and $y(2) = 5$. **SOLUTION** The differential equation is separable and $y = 1 + x^2$