2009 Conic Topic Test (Alpha)

Solutions

1.E.

2.D $x^2+4xy+4y^2-z^2=0\rightarrow(x+2y)^2-z^2=0\rightarrow(x+2y+z)(x+2y-z)=0\rightarrow x+2y+z=0, x+2y-z=0$. This is the equation of 2 planes.

3. C – setup a system of equations $\begin{matrix} a+b-5=0 \\ 4a+2b-5=7 \end{matrix}$. Solving the system produces the equation $x=y^2+4y-5$. Therefore the y-coordinate of the vertex is – 2.

4. C - $P=2r+\frac{\theta}{2\pi}(2\pi r)=2r+\theta r=10$ and $A=\frac{\theta}{2\pi}(\pi r^2)=\frac{\theta r^2}{2}=4$. Therefore $\theta=\frac{8}{r^2}$ and $2r+\frac{8}{r}=10$. Solving yields $r=1$and $r=4$. When $r=4$, $\theta=\frac{1}{2}$and when $\theta=\frac{1}{2}$, $\theta=8$. But 8 is greater than 2π, so we reject that value.

5. C - $r^2\cos 2\theta=1\rightarrow(x^2+y^2)(x^2-y^2)=1\rightarrow x^4-y^4=1$.

6. B - $c^2+b^2=a^2\rightarrow(4)^2+(2\sqrt{5})^2=a^2\rightarrow 36=a^2\rightarrow 6=a\rightarrow 12=2a$

7. D - $x^2+xy+y^2=1\rightarrow r^2\cos^2\theta+(r\cos\theta)(r\sin\theta)+r^2\sin^2\theta=1\rightarrow r^2(1+\cos\theta\sin\theta)$

8. A - $r=\frac{6}{2+\cos\theta}\rightarrow 2r+r\cos\theta=6\rightarrow 2\sqrt{x^2+y^2}+x=6\rightarrow 3(x+2)^2+4y^2=48$

9. B - $y^2 - y(4x-1) + x^2 - 2x - 12 = 0$. $y = \frac{4x-1 \pm \sqrt{49}}{2}$; $y = 2x - 1 \pm 7$; $y = 2x + 6$ and $y = 2x - 8$ parallel lines.

10. A – C,E,P,H

11. A - $\frac{|2(0)+1(0)-5|}{\sqrt{5}}=\sqrt{5}\rightarrow|r|\geq\sqrt{5}$

12. D – a hyperbola has eccentricity greater than one. Finding th ethe distinct permutations in the word hyperbola gives $9!=362880$

13. D - $R=\frac{abc}{4(Area)}=\frac{(7)(8)(9)}{4\sqrt{12(5)(4)(3)}}=\frac{21\sqrt{5}}{10}$

14. C – this the geometric definition of an ellipse. A and C are similar but our foci lie on the x –axis therefore we choose C.

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15. A - $x^2 - y^2 = 2 \rightarrow \left(\frac{x'+y'}{\sqrt{2}}\right)^2 - \left(\frac{x'-y'}{\sqrt{2}}\right)^2 = 2;\ x'y' = 1$

16. B – the area of a polygon inscribed in a circle of radius R is $A = .5(n)(R^2)\sin\left(\frac{2\pi}{n}\right)$, where n is the number of sides. Plugging in we get $A = .5(24)(13^2)\sin\left(\frac{2\pi}{24}\right) = 12 \bullet 169 \bullet \left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) = 507\left(\sqrt{6}-\sqrt{2}\right)$.

17. E - The two graphs intersect at the points $(6,6)$ and $(-1,-1)$. Using the points to find the slope and the midpoint, the equation becomes $x + y = 5$ which sums to 7.

18. B - $(x-6)^2 + (y-8)^2 = \frac{(4x+3y-4)^2}{25} \rightarrow 9x^2 - 24xy + 16y^2 - 268x - 376y + 2484 = 0 \rightarrow 1841$

19. C – since $\frac{3}{2} > 1$, then it is a dimpled limaçon.

20. B - $r^2 - 8r\left(\frac{x}{r}\right) + 6r\left(\frac{y}{r}\right) = 0 \rightarrow x^2 + y^2 - 8x + 6y = 0 \rightarrow (x-4)^2 + (y+3)^2 = 25 \rightarrow A = 25\pi$

21. C – the base has equation $\left(\frac{x+2}{3}\right)^2 + \left(\frac{y-1}{4}\right)^2 = 1$ so that base has area 12π. Therefore the volume of the cone will be $\frac{1}{3}(12\pi)(10) = 40\pi$.

22. E ($12\sqrt{3}$) – Let $a = 24$ and $b = 20$. Therefore $\frac{x^2}{576} + \frac{y^2}{400} = 1$ and the point (x , 10) is on the graph so $x^2 = 432 \rightarrow x = 12\sqrt{3}$ and double that to get the entire width of the base.

23. A - $(x-2)^2 + (y-1)^2 = 50$. Center is (2,1)

24. D - $\tan 2\theta = \frac{-12}{8-(-8)} = -\frac{3}{4}$ which makes $\cos 2\theta = -\frac{4}{5}$. Since value is close to – 1, this means this is a high 2nd quadrant angle. Dividing this angle in half makes it high 1st quadrant angle. The only logical choice would be D.

25. Omit

26. E (0,34) – plug in 0 for x and solve.

27. C – the x and y term differ by a sign therefore it is a hyperbola.

28. B – place in center radius form to find the center.

29. C – just find the vertex of the parabola.

30. B – the center of the circle lies on the perpendicular bisectors. The perpendicular bisectors are $x-y=-1$ and $4x+3y=17$. Solving the system gives the center of the circle (2,3).

Tiebreaker Answers:

1.In order to be tangent to the x – axis, you need one root and that only happen when the discriminant is zero. So k = 9.

2. Place in center-radius form and solve. The area is 8π.

3. Connecting the centers of the circles we see that it makes an equilateral triangle of side 8. The angle subtended by the arc is 120. So the length of the band would be $24+3\left(\frac{8\pi}{3}\right)=24+8\pi$.