

2009 Alpha Equations and Inequalities

Solutions

- $\frac{10x-3}{(3x-1)(4x-1)} \geq 0$; critical points: $x = 3/10, 1/3, 1/4$. Using sign chart $- + - + \rightarrow$
 $[1/4, 3/10) \cup (1/3, \infty)$. **D**
- $x(3x+2) = 1$; $3x^2 + 2x - 1 = 0$; $x = -1, 1/3$. **B**
- Eliminating z we get: $4x - 5y = 23$ and $-x + 2y = -14 \rightarrow x =$ Solving for $y \rightarrow -3, x = 2$; $2x - y = 7$. **D**
- $\frac{2-i}{1-i} + \frac{2}{2+i} = \frac{7-2i}{3-i} = \frac{23}{10} + \frac{1}{10}i$; $a + b = 12/5$. **C**
- $x + y + z = 70, 3y = 2z - 4, 2x = z + 4$, Solving: $x = 18, y = 20, z = 32$; $y = 20$; **B**
- $9 \cdot 3^{2x} - 27 \cdot 3^x - 3^x + 3 = 0$; $a = 3^x$; $9a(a-3) - (a-3)$; $(a-3)(9a-1) = 0$; $3^x = 3, 3^x = 3^{-2}$; $1 - 2 = -1$
E
- $2 - \frac{x-2}{2x-5} = 5$; $\frac{4x-10-x+2}{2x-5} = 5$; $10x - 25 = 3x - 8$; $x = 17/7$; **E**
- $\frac{|2|x|-3|}{|4|x-1|} = 2$; $1 > x \geq 0$; $\frac{|2x-3|}{|-4x+4|} = -2$; $2x - 3 = 8x - 8$; $x = 5/6$. $x \leq 0$; $\frac{|-2x-3|}{|-4x+4|} = -2$;
 $-2x - 3 = -8x + 8$; $x = 11/10 + 5/6 = \mathbf{B}$
- $x^3 + x^2 - 2x \geq 0$; $x(x+2)(x-1) \geq 0$. Using a sign chart: $--(-2)++(0)-(1)+++$; $[-2, 0] \cup [1, \infty)$. **C**
- $9(a+1)^2 + 9(a-2) + 5 = 6$; $9a^2 + 27a - 10 = 0$; $(3a+10)(3a-1)$; $a = 1/3, -10/3$. **D**
- $2\csc y(\tan y + 1) + (\tan y + 1) = 0$; $(\tan y + 1)(2\csc y + 1) = 0$. $y = \tan^{-1}(-1), 3\pi/4, 7\pi/4$;
 $\csc y \neq 1/2$. **D**
- $g(x) = ax + b$, then $g(f(x)) = a(3x^2 - 2x + 1) + b = 6x^2 - 4x - 1$; $3ax^2 - 2ax + a + b \rightarrow 3ax^2 = 6x^2$,
 $a = 2, b + 2 = -3, b = -5$; $g(x) = 2x - 3$; $g(-2) = -7$. **A**
- $\log_3(\sin 3x)^2 - \log_3(\cos 3x)^2 = 1$; $\frac{(\sin 3x)^2}{(\cos 3x)^2} = 3$; $\tan 3x = \pm\sqrt{3}$; $3x = \tan^{-1} \pm\sqrt{3}$; $3x = 60^\circ + k180^\circ$;
 $x = 20^\circ + k60^\circ$; $x = 20, 80, 140$; $3x = 120^\circ + k180^\circ$; $x = 40^\circ + k60^\circ$; $x = 40, 100, 160$. **B**
- $x = 1/c$; $\sqrt{\frac{3}{2}x + \sqrt{x}} - \sqrt{\frac{3}{2}x - \sqrt{x}} = \sqrt{2}$; $\frac{3}{2}x + \sqrt{x} + 2\sqrt{\frac{9}{4}x^2 - x} + \frac{3}{2}x - \sqrt{x} = 2$;
 $3x - 2 = 2\sqrt{\frac{9}{4}x^2 - x}$; $8x = 4, x = 1/2$; $c = 2$. **B**

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15. $s = 20$; $50\sqrt{2} = \sqrt{20(20-x)^2(-20+2x)}$; $5000 = 20(20-x)^2(-20+2x)$;
 $125 = (20-x)^2(-20+2x)$; $5 = 20-x$; $x = 15$, $y = 10$. **C**

16. $2(-\sqrt{2})^2 - 3(a-2\sqrt{2})(-\sqrt{2}) + 5 = 0$; $4 + 3\sqrt{2}(a-2\sqrt{2}) + 5 = 0$; $9 - 12 + 3\sqrt{2} = 0$; $x = \sqrt{2}/2$. **A**

17. $\frac{|-2(2)-1(1)+1|}{\sqrt{4+1}} = 4/\sqrt{5}$; $r = 4\sqrt{5}/5$. **D**

18. $(500 + 100x)(40 - 5x) = R(x)$; $x = \frac{8-5}{2} = 1.5$. Increase of \$150 or Rent at \$650. **C**

19. $x^4 + x^3 + x + 1 = 0$; $(x^3 + 1)(x + 1) = 0$; $x = -1$. Undefined at $x = 1$, -1 so \emptyset . **D**

20. $\frac{3x+5}{y} = 10$, $3x - 10y = -5$; $5x - 2y = 6 \rightarrow x = 35/22$; $y = 43/44$; **D**

21. $(k + l + m + n)^2 = 100 = k^2 + l^2 + m^2 + n^2 + 2(kl + km + kn + lm + ln + mn) \rightarrow 100 - 70 = 30$. **C**

22. $x^2 - x - 2 = 0$; $x = 2, -1$; $(2, 6), (-1, 9)$. $d = \sqrt{9+9} = 3\sqrt{2}$. **B**

23. $a = 1/x$, $b = 1/y$; $3a + 5b = 10$ and $2a + 3b = 9$; $a = 15$, $x = 1/15$; $b = -7$, $y = -1/7$; $2 + 3 = 5$. **D**

24. $\frac{Sx+3S+Tx-2T}{(x-2)(x+3)} = \frac{8x-1}{x^2+x-6}$; $S + T = 8$, $3S - 2T = -1$, $(3, 5)$; $3 + 5 = 8$. **E**

25. $(3 + 2i)(3 - 2i) + 2(3 + 2i) - 3(3 - 2i) = 10 + 10i$; **C**

26. $P = 1 + \frac{1}{x}$; $x = 100$ then $x = 1 + .01 = 1.01$; $x = .01$; $x = .01$ then $P = 101$. **A**

27. $6a^2 - 35a + 50 = 0$; $(3a - 10)(2a - 5) = 0$, Substituting: $x = 1/2, 2, 1/3, 3$. **B**

28. $\begin{vmatrix} x & -1 & 2 \\ 2 & x & 4 \\ 0 & 3 & 1 \end{vmatrix} = 2$; $x(x - 12) - 2(-7) = 2$; $x^2 - 12x + 12 = 0$; **D**

29. $y = \frac{97-5x}{11}$; $(4, 7), (15, 2)$. 2 positive integer pairs. **C**

30. $8(\log_3 x)^2 = 14\log_3 x + 3$; $(2\log_3 x - 3)(4\log_3 x - 1) = 0$; $3^{3/2} + 3^{1/4} = 3\sqrt{3} + \sqrt[4]{3}$. **C**

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Tie-Breakers:

1. $1 \leq |2x - 1| \leq 5$; $2 \leq 2x \leq 6$; $1 \leq x \leq 3$ and $-1 \geq 2x - 1 \geq -5$; $0 \geq x \geq -2$. $[-2, 0] \cup [1, 3]$.

2. $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & 1 \end{bmatrix} = \begin{bmatrix} a & 2a + 3b & -a + b \\ c & 2c + 3d & -c + d \end{bmatrix} = \begin{bmatrix} 2 & -5 & -5 \\ 4 & 23 & 1 \end{bmatrix}$, $a = 2$, $b = -3$, $c = 4$, $d = 5$. $\begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$

3. Points of change: $(-1, 3)$ and $(3, 3)$. Length of base of trapezoid: 4, $h = 4$, $7 = 2x - 1$, $(3, 7)$
 $-x - 1 - x + 2 = 7$, $(-3, 7)$. Length of 2nd base = 6, $A = 20$.