2009 Logs and Exponents (Alpha)

Solutions

1. $3 = 2\log_2 x$; $\frac{3}{2} = \log_2 x$; $2^{\frac{3}{2}} = x \rightarrow$ $\mathbf{2\sqrt{2}}$ **b)**

2. $3^{x+1} \cdot 2^{2y-x} = 2^4 \cdot 3^2 \rightarrow x + 1 = 2$ and $2y - x = 4$. Solving $x = 1$, $y = {}^5/_2$. Sum = ${}^7/_2$ **a)**

3. $\log_b c = d \rightarrow c = b^d$; $1 = \log_c b^d$; $\frac{1}{d} = \log_c b \rightarrow 3\log_c b + 2 \rightarrow 3\left(\frac{1}{d}\right) + 2 \rightarrow$ $\mathbf{\frac{3+2d}{d}}$ **d)**

4. $\log_2 x = 3 - \log_2(x + 2) \rightarrow \log_2 x + \log_2(x + 2) = 3$; $\log_2 x(x + 2) = 3$; $x^2 + 2x - 8 = 0$; $\boldsymbol{x = 2, \neq -4}$ **b)**

5. $2\log 3x - \log x^2 + \log(x + 1) \rightarrow \log 9x^2(x + 1) - \log x^2 \rightarrow \mathbf{\log 9(}\boldsymbol{x}\mathbf{ + 1)}$ **e)**

6. $\log_2 8 - \log_{.5} 4 + \log_{.25} 16 - \log_{\sqrt{2}}\left(\frac{1}{16}\right) \rightarrow 2^x = 2^3$, $2^{-x} = 2^2$, $2^{-2x} = 2^4$, $2^{-\frac{1}{2}x} = 2^{-4} \rightarrow 3 - (-2) - 2 - 8 = -5$
d)

7. $9^x - 2(3^{x+1}) + 8 = 0 \rightarrow 3^{2x} - 6 \cdot 3^x + 8$; $(3^x - 4)(3^x - 2) = 0$; $3^x = 4,2$; $x = \log_3 4 + \log_3 2 \rightarrow \boldsymbol{\log_3 8}$ **d)**

8. $\log_2 x + \log_4 x + \log_{16} x = 7 \rightarrow \log_2 x + \log_4 2 \cdot \log_2 x + \log_{16} 2 \cdot \log_2 x = 7$; $\log_2 x\left(1 + \frac{1}{2} + \frac{1}{4}\right) = 7$
$\log_2 x = \frac{4}{7}(7) = 4$; $\boldsymbol{x = 16}$ **e)**

9. I: $y = 4^{3x-2} \rightarrow y = 2^{6x-4}$; II: $y = 2(2^{3x-2}) \rightarrow y = 2^{3x-1}$; III: $y = 2^{3x-1}$ **II, III** **c)**

10. $y = (\log_2 3)(\log_3 4)\cdots(\log_n[n + 1])\cdots(\log_{31} 32) \rightarrow \frac{\log_2 3}{\log_2 2} \cdot \frac{\log_2 4}{\log_2 3} \cdot \frac{\log_2 5}{\log_2 4} \cdots \frac{\log_2(n+1)}{\log_2 n} \cdots \frac{\log_2 32}{\log_2 31} = \frac{\log_2 32}{\log_2 2} = \mathbf{5}$
b)

11. $g(t) = P(1 + r)^t \rightarrow 2 \cdot 174000 = 174000(1.025)^t$; $\log 2 = t\log(1.025)$; $\boldsymbol{t} = \frac{\boldsymbol{\log 2}}{\mathbf{\log(1.025)}} \rightarrow {}^{.301}/_{.011} \approx 27$ **a)**

12. $f(x) = \log_2 x$; $g(x) = 2\log_2(x - 1) - 2$. To return $(x - 1)$ to $x \rightarrow +1$ (shift to the right), to return -2 to 0 → add 2 (shift up), In $2\log(x - 1) - 2$ every y value is doubled (stretch). **c)**

13. $f(k) = e^{k-1} - 3 \rightarrow \frac{e^k}{e} - 3 = 0$; $e^k = 3e$; $k = \ln 3e \rightarrow$ $\boldsymbol{\ln 3 + 1}$ **d)**

14. $\log(x + 4) - \log(2x - 3) = \log 2 \rightarrow \log\frac{x+4}{2x-3} = \log 2 \rightarrow x + 4 = 4x - 6$, $\boldsymbol{x} = \mathbf{\frac{10}{3}}$ **a)**

15. $\log 2 = a$, $\log 3 = b$, $\log 5 = c$. express $\log 450$ in terms of a, b, and c. $\log 450 = \log(2 \cdot 3^2 \cdot 5^2)$
$\log 2 + 2\log 3 + 2\log 5 = \boldsymbol{a + 2b + 2c}$ **c)**

16. $\frac{\log_2(x+1)}{\log_2(2x-3)} \leq 1 \rightarrow \frac{\log_2(x+1)}{\log_2(2x-3)} - 1 \leq 0$; $\frac{\log_2(x+1) - \log_2(2x-3)}{\log_2(2x-3)} \leq 0$; $\frac{\log_2\frac{(x+1)}{(2x-3)}}{\log_2(2x-3)} \leq 0$;
Case I: $\log_2\frac{(x+1)}{(2x-3)} \leq 0$ and $\log_2(2x - 3) > 0$; $x \in (3/2, 2)$
Case II: $\log_2\frac{(x+1)}{(2x-3)} \geq 0$ and $\log_2(2x - 3) < 0$; $x \in [4, \infty) \rightarrow (3/2, 2) \cup [4, \infty)$. **b)**

17. $128^{(x-1)} \to 2^{(7x-7)}$; $4^{(2x+3)} \to 2^{(4x+6)}$; $32^x \to 2^{5x}$, $\frac{2^{7x-7}}{2^{4x+6}} = 2^{5x} \to$ $3x - 13 = 5x$; $x = -\frac{13}{2}$; $\frac{\mathbf{x}}{\mathbf{2}} = -\frac{\mathbf{13}}{\mathbf{4}}$ **d)**

18. $\log_x 9 + \log_9 x = \frac{10}{3}$; $\frac{\log 9}{\log x} + \frac{\log x}{\log 9} = \frac{10}{3}$; $3(\log 9)^2 + 3(\log x)^2 = 10 (\log 9)(\log x) \to$
$(3\log x - \log 9)(\log x - 3\log 9) = 0 \to \log x^3 = \log 3^2$, $\log x = \log 3^6$; $x = 3^{\frac{2}{3}}$; **729 c)**

19. $\log_2 .25 + \log_{.5} 32 - \log_{\sqrt{2}} 4 \to 2^x = \frac{1}{4}$, $x = -2$; $.5^x = 2^5$, $x = -5$; $2^{\frac{1}{2}x} = 2^2$, $x = 4$, **Sum = -11 b)**

20. $P(t) = \frac{800}{1+49e^{-0.2t}}$; $P(t)(1 + 49e^{-0.2t}) = 800$, $P(t) + P(t) \cdot 49e^{-.2t} = 800$, $e^{-.2t} = \frac{800 - P(t)}{49P(t)}$,
$-.2t = \ln\left(\frac{800-P(t)}{49P(t)}\right) \to \boldsymbol{t = -5 \ln\left(\frac{800-P(t)}{49P(t)}\right)}$ **e)**

21. $3^x \cdot 9^{2x-1} = 3^{1/2} (81)^{-1} \to 3^x \cdot 3^{4x-2} = 3^{1/2}(3)^{-4}$, $5x - 2 = -\frac{7}{2}$, $\boldsymbol{x = -\frac{3}{10}}$. **a)**

22. $\log_3 x = (-2 + \log_2 100)(\log_3 \sqrt{2})$; $\log_3 x = (-2 + \log_2 4 \cdot 25)(\log_3 \sqrt{2})$; $\log_3 x = \log_3 \sqrt{2}^{\log_2 25} \to$
$\log_3 x = \log_3 5$; $\boldsymbol{x = 5}$. **d)**

23. $\log_x \frac{ab^3\sqrt{ac}}{c^2} = \log_x ab^3 + \frac{1}{2} \log_x ac - 2\log_x c = 3\log_x b + \log_x a + \frac{1}{2}(\log_x a + \log_x c) - 10 \to$
$-9 + 2 + 1 + 5/2 - 10 = -13.5$ **b)**

24. $\log_{10} \frac{x}{y} = 1$, $\log_{10} y^x = 100$; $\frac{x}{y} = 10 \to x = 10y$, $10^{100} = y^x$; $10^{100} = y^{10y} \to 10^{10} = y^y$. $y = 10$, $x = 100$,
$2x - 3y = 170$. **b)**

25. $(3^4)^{\cos^2 x} + (3^4)^{\sin^2 x} = 30$. $(3^4)^{\cos^2 x} - (3^4)^{1-\cos^2 x} - 30 = 0 \to (3^4)^{\cos^2 x} \cdot (3^4)^{\cos^2 x} - 30(3^4)^{\cos^2 x} - 3^4$
$\left((3^4)^{\cos^2 x} - 27\right)\left((3^4)^{\cos^2 x} - 3\right) = 0$, $4\cos^2 x = 1$; $\cos x = \pm\frac{1}{2}$, $\cos x = \pm\frac{\sqrt{3}}{2} \to \cot x = \boldsymbol{\pm\sqrt{3}, \pm\frac{\sqrt{3}}{3}}$ **c)**

26. $\sum_7^{2008} \log_7 \left(\frac{n}{n+1}\right) = \log_7 \frac{7}{8} + \log_7 \frac{8}{9} + \log_7 \frac{9}{10} \ldots + \log_7 \frac{2007}{2008} + \log_7 \frac{2008}{2009} \to \log_7 \frac{7}{8} \cdot \frac{8}{9} \cdot \frac{9}{10} \ldots \cdot \frac{2007}{2008} \cdot \frac{2008}{2009}$
$\log_7 7 - \log_7 49 - \log_7 41 = -1 - \log_7 41$. $-3 - 82 = -85$. ***e***

27. $4^{3m-1} = 1 + 8^{2(k+3)}$; $2^{6m-2} - 2^{6x+18} = 1 \to 6m - 2 = 1$, $m = \frac{1}{2}$; $6k + 18 = 0$, $k = -3 \to \left(\frac{1}{2}\right)^{-3} = {}^{1}/_{8}$. **e)**

28. $½\log_5(x - 2) = 3\log_5 2 - \frac{3}{2}\log_5(x - 2) \to \log_5(x - 2) = 6\log_5 2 - 3\log_5(x - 2)$;
$\log_5(x - 2) - \log_5 64 + \log_5(x - 2)^3 = 0 \to \log_5\frac{(x-2)^4}{64} = 0 \to (x - 2)^4 = 64$; $x = 2 \pm 2\sqrt{2} \to \mathbf{2 + 2\sqrt{2}}$.
c

29. Shift to the left $\to e^{x+1}$, vertical shift of -3 $\to e^{x+1} - 3$, vertical shrink of 2 $\to 2e^{x+1} - 3$ **e)**

30. $\log_a x + \log_{a^2} x + \log_{a^4} x \rightarrow \log_a x + \log_{a^2} a \cdot \log_a x + \log_{a^4} a \cdot \log_a x = c$; $\log_a x\left(1 + \frac{1}{2} + \frac{1}{4}\right) = c$

$\log_a x\left(\frac{7}{4}\right) = c$; $\log_a x = \left(\frac{4}{7}\right)c$; $x = a^{\frac{4}{7}c}$ **b)**

Tie-Breakers:

1. $(\sqrt{13} + \sqrt{3})(\sqrt{13} - \sqrt{3}) = 13 - 3 = 10$; $\log_{10}(\sqrt{13} + \sqrt{3})(\sqrt{13} - \sqrt{3}) = \log_{10}(\sqrt{13} + \sqrt{3}) + \log_{10}(\sqrt{13} - \sqrt{3}) = \log_{10} 10 \rightarrow \log_{10}(\sqrt{13} - \sqrt{3}) = 1 - a$.

2 $f(x) = \log_2(x^2) - \log_2(3x) - 5$; $f(3) = \log_2(9) - \log_2(9) - 5 \rightarrow 5$

3. $2^{3\log_2 5} \cdot 9^{\log_3 x} = 2^{\log_2 125} \cdot 3^{\log_3 x^2} = 125x^2$.