2009 Alpha Logs & Exponents

E is none of these

1. Find the zero(s) of the graph of $y = 2log_2x - 3$.

a) 3 b) $2\sqrt{2}$ c) 6 d) 9

2. $3^{x+1} \cdot 2^{2y-x} = 144$. Find the sum of $x + y$.

a) $7/2$ b) $5/2$ c) 2 d) 1

3. If $log_b c = d$ then $3log_c b + 2$ equals:

a) $3d + 2$ b) $d^3 + 2$ c) $\frac{3d}{d+2}$ d) $\frac{3+2d}{d}$

4. Solve for x if $log_2x = 3 - log_2(x + 2)$

a) 1 b) 2 c) 3 d) 4

5. $2log\ 3x - log\ x^2 + log(x + 1)$ when simplified equals $log\ w$, find w.

a) $\frac{3(x-1)}{2x}$ b) $1 + 7x - x^2$ c) $\frac{9(x+1)}{x^2}$ d) $\frac{9}{x+1}$

6. Find the value of $log_2 8 - log_{.5} 4 + log_{,25} 16 - log_{\sqrt{2}}\left(\frac{1}{16}\right)$

a) 16 b) 4 c) - 4 d) - 5

7. The sum of the solutions to: $9^x - 2(3^{x+1}) + 8 = 0$ is:

a) $log_3 2$ b) $log_3 4$ c) $log_3 6$ d) $log_3 8$

8. Find the real value of x that satisfies $log_2 x + log_4 x + log_{16} x = 7$

a) 2 b) 4 c) 8 d) 32

9. Which of the following are equivalent: I: $y = 4^{3x-2}$, II: $y = 2(2^{3x-2})$, III: $y = 2^{3x-1}$

a) I, II b) I, III c) II, III d) I, II, III

10. If $y = (log_2 3)(log_3 4)\cdots(log_n[n + 1])\cdots(log_{31}32)$ then

a) $4 < y < 5$ b) $y = 5$ c) $5 < y < 6$ d) 6

11. Population growth is modeled by $g(t) = P(1 + r)^t$. Knoxville has a population of 174,000 and has a yearly growth rate of 2.5%. Predict in how many years(to the nearest year)will the population double? ($\log 2$ =.301),($\log 1.025$=.011), ($\log 174000 = 5.24$)

a) 27 b) 26 c) 25 d) 24

12. Given $f(x) = \log_2 x$. Describe the transformation of the graph $f(x)$ to the graph of $g(x) = 2\log_2(x - 1) - 2$.

a) ↑ 2, ← 1, stretch 2 b) ↓ 2, ← 1, stretch 2 c) ↓ 2, → 1, stretch 2 d) c) ↓ 2, → 1, shrink 2

13. $f(k) = e^{k-1} - 3$. What is the zero of this function?

a) 0 b) $\ln 3 - 1$ c) $3e$ ` d) $\ln 3 + 1$

14. Solve: $\log(x + 4) - \log(2x - 3) = \log 2$

a) $\frac{10}{3}$ b) 4 c) $\frac{14}{3}$ ` d) 7

15. $\log 2 = a$, $\log 3 = b$, $\log 5 = c$. express $\log 450$ in terms of a, b, and c.

a) $2(a + b + c)$ b) ab^2c^2 ` c) $a + 2b + 2c$ d) $(abc)^2$

16. Solve: $\frac{\log_2(x+1)}{\log_2(2x-3)} \leq 1$

a) $x \leq 4$ b) $(3/2, 2) \cup [4, \infty)$. c) $(3/2, 2) \cup (2, 4]$ d) $\emptyset$

17. If $128^{(x-1)}$ divided by $4^{(2x+3)}$ equals 32^x, find the value of $\frac{x}{2}$.

a) 2 b) $\frac{17}{6}$ c) $-\frac{13}{6}$ d) $-\frac{13}{4}$

18. Find the greatest value of x that solves $\log_x 9 + \log_9 x = \frac{10}{3}$.

a) 27 b) 8 c) 729 d) $3\sqrt{3}$

19. Find the value of $\log_2 .25 + \log_{.5} 32 - \log_{\sqrt{2}} 4$.

a) -3 b) -11 c) 0 d) 2

20. The number of students infected with the flu at the local high school is modeled by: $P(t) = \frac{800}{1+49e^{-0.2t}}$. Solve for t.

a) $t = 50\ ln\left(\frac{800 + P(t)}{49}\right)$
b) $t = -50\ ln\left(\frac{800-P(t)}{49}\right)$
c) $t = 49\ ln\left(\frac{800-P(t)}{50}\right)$
d) $t = -49\ ln\left(\frac{800+P(t)}{50}\right)$

21. Find the solution to $3^x \cdot 9^{2x-1} = 3^{1/2}(81)^{-1}$.

a) $-\frac{3}{10}$ b) $-\frac{1}{2}$ c) $\frac{1}{6}$ d) $\frac{2}{5}$

22. Find the value of x that satisfies $log_3 x = (-2 + log_2 100)(log_3 \sqrt{2})$.

a) $log_3 10\sqrt{2}$ b) 20 c) -3 d) 5

23. Let $log_x a = 2$, $log_x b = -3$ and $log_x c = 5$; find the value of $log_x \frac{ab^3\sqrt{ac}}{c^2}$.

a) $^{18}/_5$ b) - 13.5 c) $-27\sqrt{10}$ d) $^{54}/_{25}$

24. If $log_{10} x - log_{10} y = 1$ and $xlog_{10}y = 100$, find $2x - 3y$.

a) 160 b) 170 c) 180 d) 200

25. Find $cot\ x$ where $0 < x < \pi$ if $(3^4)^{cos^2 x} + (3^4)^{sin^2 x} = 30$.

a) $\pm\frac{2\sqrt{3}}{3}, \pm 2$ b) $\frac{2\sqrt{3}}{3}, -2$ c) $\pm\frac{\sqrt{3}}{3}, \pm\sqrt{3}$ d) $\pm 1, 0$

26. If $\sum_7^{2008} log_7\left(\frac{n}{n+1}\right) = a + blog_7 c$, find $3a - 2c$.

a) -505 b) 499 c) -1001 d) 998

27. If $4^{3m-1} = 1 + 8^{2(k+3)}$, find m^k.

a) 27 b) 21 c) 9 d) 3

28. Solve: $\frac{1}{2}log_5(x-2) = 3log_5 2 - \frac{3}{2}log_5(x-2)$

a) $-2 + 2\sqrt{2}$ b) $2\sqrt{2}$ c) $2 + 2\sqrt{2}$ d) $\sqrt{2}$

29. If $f(x) = e^x$ which of the following is $g(x)$ after a shift to the left of 1 a vertical shift of -3 and a shrink of 2

a) $g(x) = 2e^{x-1} + 3$ b) $g(x) = \frac{1}{2}e^{x-1} - 3$ c) $g(x) = 2e^{x+1} + 3$ d) $g(x) = \frac{1}{2}e^{x+1} - 3$

30. If $log_a x + log_{a^2} x + log_{a^4} x = c$ find x in terms of c.

a) $\frac{7}{4}c$ b) $a^{\frac{7}{4}c}$ c) $a^{\frac{7}{4c}}$ d) $c^{\frac{7}{4}}$

Tie-Breakers:

1. $a = log_{10}(\sqrt{13} + \sqrt{3})$. Express in terms of a the value of $log_{10}(\sqrt{13} - \sqrt{3})$.

2. Find $f(3)$ if $f(x) = log_2 (x^2) - log_2 (3x) - 5$.

3. Simplify: $2^{3log_2 5}$.

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Solutions

1. $3 = 2log_2x$; $\frac{3}{2} = log_2 x$; $2^{\frac{3}{2}} = x \rightarrow$ $\mathbf{2\sqrt{2}}$ **b)**

2. $3^{x+1} \cdot 2^{2y-x} = 2^4 \cdot 3^2 \rightarrow x + 1 = 2$ and $2y - x = 4$. Solving $x = 1$, $y = {}^5/_2$. Sum $= {}^7/_2$ **a)**

3. $log_b c = d \rightarrow c = b^d$; $1 = log_c b^d$; $\frac{1}{d} = log_c b \rightarrow 3log_c b + 2 \rightarrow 3\left(\frac{1}{d}\right) + 2 \rightarrow \mathbf{\frac{3+2d}{d}}$ **d)**

4. $log_2x = 3 - log_2(x + 2) \rightarrow log_2x + log_2(x + 2) = 3$; $log_2 x(x + 2) = 3$; $x^2 + 2x - 8 = 0$; $\boldsymbol{x = 2, \neq -4}$ **b)**

5. $2log\ 3x - log\ x^2 + log(x + 1) \rightarrow log\ 9x^2(x + 1) - log\ x^2 \rightarrow \mathbf{log\ 9(\boldsymbol{x} + 1)}$ **e)**

6. $\log_2 8 - log_{.5}\ 4 + log_{.25}\ 16 - log_{\sqrt{2}}\left(\frac{1}{16}\right) \rightarrow 2^x = 2^3$, $2^{-x} = 2^2$, $2^{-2x} = 2^4$, $2^{-\frac{1}{2}x} = 2^{-4} \rightarrow 3 - (-2) - 2 - 8 = -5$ **d)**

7. $9^x - 2(3^{x+1}) + 8 = 0 \rightarrow 3^{2x} - 6 \cdot 3^x + 8$; $(3^x - 4)(3^x - 2) = 0$; $3^x = 4{,}2$; $x = log_3 4 + log_3 2 \rightarrow \boldsymbol{log_3 8}$ **d)**

8. $log_2\ x + log_4 x + log_{16} x = 7 \rightarrow log_2\ x + log_4 2 \cdot log_2 x + log_{16}\ 2 \cdot log_2 x = 7$; $\log_2 x\left(1 + \frac{1}{2} + \frac{1}{4}\right) = 7$
$log_2 x = \frac{4}{7}(7) = 4$; $\boldsymbol{x = 16}$ **e)**

9. I: $y = 4^{3x-2} \rightarrow y = 2^{6x-4}$; II: $y = 2(2^{3x-2}) \rightarrow y = 2^{3x-1}$; III: $y = 2^{3x-1}$ **II, III** **c)**

10. $y = (log_2 3)(log_3 4)\cdots(log_n[n + 1])\cdots(log_{31} 32) \rightarrow \frac{log_2 3}{log_2 2} \cdot \frac{log_2 4}{log_2 3} \cdot \frac{log_2 5}{log_2 4} \cdots\cdot \frac{log_2(n+1)}{log_2 n} \cdots\cdot \frac{log_2 32}{log_2 31} = \frac{log_2 32}{log_2 2} = \mathbf{5}$ **b)**

11. $g(t) = P(1 + r)^t \rightarrow 2 \cdot 174000 = 174000(1.025)^t$; $log\ 2 = tlog(1.025)$; $\boldsymbol{t} = \frac{\boldsymbol{log 2}}{\mathbf{log(1.025)}} \rightarrow {}^{.301}/_{.011} \approx 27$ **a)**

12. $f(x) = log_2\ x$; $g(x) = 2log_2\ (x - 1) - 2$. To return $(x - 1)$ to $x \rightarrow +1$ (shift to the right), to return -2 to 0 → add 2 (shift up), In $2log(x - 1) - 2$ every y value is doubled (stretch). **c)**

13. $f(k) = e^{k-1} - 3 \rightarrow \frac{e^k}{e} - 3 = 0$; $e^k = 3e$; $k = ln\ 3e \rightarrow$ $\boldsymbol{ln\ 3 + 1}$ **d)**

14. $log(x + 4) - log(2x - 3) = log\ 2 \rightarrow log\frac{x+4}{2x-3} = log\ 2 \rightarrow x + 4 = 4x - 6$, $\boldsymbol{x} = \mathbf{\frac{10}{3}}$ **a)**

15. $log\ 2 = a$, $log\ 3 = b$, $log\ 5 = c$. express $log\ 450$ in terms of a, b, and c. $Log\ 450 = log\ (2 \cdot 3^2 \cdot 5^2)$
$log\ 2 + 2log\ 3 + 2\ log\ 5 = \boldsymbol{a + 2b + 2c}$ **c)**

16. $\frac{log_2(x+1)}{log_2(2x-3)} \leq 1 \rightarrow \frac{log_2(x+1)}{log_2(2x-3)} - 1 \leq 0$; $\frac{log_2(x+1) - log_2(2x-3)}{log_2(2x-3)} \leq 0$; $\frac{log_2\frac{(x+1)}{(2x-3)}}{log_2(2x-3)} \leq 0$;
Case I: $log_2\frac{(x+1)}{(2x-3)} \leq 0$ and $log_2(2x - 3) > 0$; $x \in (3/2, 2)$
Case II: $log_2\frac{(x+1)}{(2x-3)} \geq 0$ and $log_2(2x - 3) < 0$; $x \in [4, \infty) \rightarrow (3/2, 2) \cup [4, \infty)$. **b)**

17. $128^{(x-1)} \to 2^{(7x-7)}$; $4^{(2x+3)} \to 2^{(4x+6)}$; $32^x \to 2^{5x}$, $\frac{2^{7x-7}}{2^{4x+6}} = 2^{5x} \to$ $3x - 13 = 5x$; $x = -\frac{13}{2}$; $\frac{\mathbf{x}}{\mathbf{2}} = -\frac{\mathbf{13}}{\mathbf{4}}$ **d)**

18. $log_x 9 + log_9 x = \frac{10}{3}$; $\frac{log 9}{log x} + \frac{log x}{log 9} = \frac{10}{3}$; $3(log\ 9)^2 + 3(log\ x)^2 = 10\ (log\ 9)(log\ x) \to$
$(3log\ x - \log 9)(\log x - 3\log 9) = 0 \to log\ x^3 = log\ 3^2$, $log\ x = log\ 3^6$; $x = 3^{\frac{2}{3}}$; **729 c)**

19. $log_2 .25 + log_{.5} 32 - log_{\sqrt{2}} 4 \to 2^x = \frac{1}{4}$, $x = -2$; $.5^x = 2^5$, $x = -5$; $2^{\frac{1}{2}x} = 2^2$, $x = 4$, **Sum = -11 b)**

20. $P(t) = \frac{800}{1+49e^{-0.2t}}$; $P(t)(1 + 49e^{-0.2t}) = 800$, $P(t) + P(t) \cdot 49e^{-.2t} = 800$, $e^{-.2t} = \frac{800 - P(t)}{49P(t)}$,
$-.2t = ln\left(\frac{800-P(t)}{49P(t)}\right) \to \boldsymbol{t = -5\ ln\left(\frac{800-P(t)}{49P(t)}\right)}$ **e)**

21. $3^x \cdot 9^{2x-1} = 3^{1/2}(81)^{-1} \to 3^x \cdot 3^{4x-2} = 3^{1/2}(3)^{-4}$, $5x - 2 = -\frac{7}{2}$, $\boldsymbol{x = -\frac{3}{10}}$. **a)**

22. $log_3 x = (-2 + log_2 100)(\log_3 \sqrt{2})$; $log_3 x = (-2 + log_2 4 \cdot 25)(log_3 \sqrt{2})$; $log_3 x = log_3 \sqrt{2}^{log_2 25} \to$
$log_3 x = \log_3 5$; $\boldsymbol{x = 5}$. **d)**

23. $log_x \frac{ab^3\sqrt{ac}}{c^2} = log_x ab^3 + \frac{1}{2} log_x ac - 2log_x c = 3log_x b + log_x a + \frac{1}{2}(log_x a + log_x c) - 10 \to$
$-9 + 2 + 1 + 5/2 - 10 = -13.5$ **b)**

24. $\log_{10} \frac{x}{y} = 1$, $\log_{10} y^x = 100$; $\frac{x}{y} = 10 \to x = 10y$, $10^{100} = y^x$; $10^{100} = y^{10y} \to 10^{10} = y^y$. $y = 10$, $x = 100$,
$2x - 3y = 170$. **b)**

25. $(3^4)^{cos^2 x} + (3^4)^{sin^2 x} = 30$. $(3^4)^{cos^2 x} - (3^4)^{1-cos^2 x} - 30 = 0 \to (3^4)^{cos^2 x} \cdot (3^4)^{cos^2 x} - 30(3^4)^{cos^2 x}$ - 3^4
$\left((3^4)^{cos^2 x} - 27\right)\left((3^4)^{cos^2 x} - 3\right) = 0$, $4cos^2 x = 1$; $cos\ x = \pm\frac{1}{2}$, $cos\ x = \pm\frac{\sqrt{3}}{2} \to cot\ x = \pm\sqrt{\mathbf{3}}, \pm\frac{\sqrt{\mathbf{3}}}{\mathbf{3}}$ **c)**

26. $\sum_7^{2008} log_7\left(\frac{n}{n+1}\right) = log_7 \frac{7}{8} + log_7 \frac{8}{9} + log_7 \frac{9}{10} \ldots + log_7 \frac{2007}{2008} + log_7 \frac{2008}{2009} \to log_7 \frac{7}{8} \cdot \frac{8}{9} \cdot \frac{9}{10} \ldots \cdot \frac{2007}{2008} \cdot \frac{2008}{2009}$
$log_7 7 - log_7 49 - log_7 41 = -1 - log_7 41$. $-3 - 82 = -85$. ***e***

27. $4^{3m-1} = 1 + 8^{2(k+3)}$; $2^{6m-2} - 2^{6x+18} = 1 \to 6m - 2 = 1$, $m = \frac{1}{2}$; $6k + 18 = 0$, $k = -3 \to \left(\frac{1}{2}\right)^{-3} = {}^1/_8$. **e)**

28. $½log_5(x-2) = 3log_5 2 - \frac{3}{2}log_5(x-2) \to log_5(x-2) = 6log_5 2 - 3log_5(x-2)$;
$log_5(x-2) - log_5 64 + log_5(x-2)^3 = 0 \to log_5\frac{(x-2)^4}{64} = 0 \to (x-2)^4 = 64$; $x = 2 \pm 2\sqrt{2} \to \mathbf{2 + 2\sqrt{2}}$.
c

29. Shift to the left $\to e^{x+1}$, vertical shift of -3 $\to e^{x+1} - 3$, vertical shrink of 2 $\to 2e^{x+1} - 3$ **e)**

30. $log_a x + log_{a^2} x + log_{a^4} x \to log_a x + log_{a^2} a \cdot log_a x + log_{a^4} a \cdot log_a x = c$; $log_a x\left(1 + \frac{1}{2} + \frac{1}{4}\right) = c$
$log_a x\left(\frac{7}{4}\right) = c$; $log_a x = \left(\frac{4}{7}\right)c$; $x = a^{\frac{4}{7}c}$ **b)**

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Tie-Breakers:

1. $(\sqrt{13}+\sqrt{3})(\sqrt{13}-\sqrt{3}) = 13-3=10$; $log_{10}(\sqrt{13}+\sqrt{3})(\sqrt{13}-\sqrt{3}) = log_{10}(\sqrt{13}+\sqrt{3}) + log_{10}(\sqrt{13}-\sqrt{3}) = log_{10} 10 \rightarrow log_{10}(\sqrt{13}-\sqrt{3}) = 1-a$.

2 $f(x) = log_2(x^2) - log_2(3x) - 5; f(3) = log_2(9) - log_2(9) - 5 \rightarrow 5$

3. $2^{3log_2 5} \cdot 9^{log_3 x} = 2^{log_2 125} \cdot 3^{log_3 x^2} = 125x^2$.