2009 Matricies and Vectors (Alpha)

Solutions

1.d – find the cross product of the two vectors which gives $-10i-2j-14k$, which reduces to $5i+j+k$.

2. A - $\sin\theta\begin{vmatrix}1 & \cos\theta\\ 1 & 1\end{vmatrix}-(-\sin\theta)\begin{vmatrix}\sin\theta & \cos\theta\\ -1 & 1\end{vmatrix}+(1-\sin\theta)\begin{vmatrix}\sin\theta & 1\\ -1 & 1\end{vmatrix}=$
$\sin\theta(1-\cos\theta)+\sin\theta(\sin\theta+\cos\theta)+(1-\sin\theta)(\sin\theta+1)=1+\sin\theta$.

3. A - $A_{old}=0.5\begin{vmatrix}3 & 2 & 1\\ 5 & 7 & 1\\ 11 & 2 & 1\end{vmatrix}=20$ and $A_{new}=0.5\begin{vmatrix}-8 & 18 & 1\\ -17 & 41 & 1\\ -24 & 50 & 1\end{vmatrix}=40$, so the difference is 20.

4. B - $\cos\theta=\dfrac{3\bullet 11+7\bullet 12+(-9)13}{\left(\sqrt{3^2+7^2+9^2}\right)\left(\sqrt{11^2+12^2+13^2}\right)}=0\rightarrow 90^\circ$

5. A- $(30+18)i+\left(0+18\sqrt{3}\right)j=48i+18\sqrt{3}j\rightarrow 6\left(8i+3\sqrt{3}j\right)=6\sqrt{64+27}=6\sqrt{91}$

6. D - $\begin{vmatrix}3 & -1 & 1\\ x & x & 6\\ x & 1 & -2\end{vmatrix}=6\rightarrow -x^2-13x-18=6\rightarrow x^2+13x+24=0\rightarrow (13)^2-2(24)=169-48-121$

7. A - $(a\bullet c)b=(21)(3i-4j)\rightarrow\sqrt{21^2(25)}=21\bullet 5=105$

8. A – note that $M^2=\begin{bmatrix}2 & 3\\ -1 & -2\end{bmatrix}\begin{bmatrix}2 & 3\\ -1 & -2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$. Therefore, simplifying the equation in question

we have $3I+M=\begin{bmatrix}5 & 3\\ -1 & 1\end{bmatrix}$. The sum of these elements is 8.

9.d – solving for z gives $z=\dfrac{\begin{vmatrix}3 & 2 & -4\\ 2 & 1 & 3\\ 1 & 1 & 5\end{vmatrix}}{\begin{vmatrix}3 & 2 & -1\\ 2 & 1 & 1\\ 1 & 1 & 2\end{vmatrix}}=\dfrac{-12}{-4}=3$.

10. D - , $\langle A+3B,-A-2B\rangle=\langle 1,2\rangle\rightarrow\langle A,B\rangle=\langle -8,3\rangle=(a+b)^2=25$

11. C – properties of matrices and their determinants.

12. C - $3(-1)-2(4)+2(1)=-9 \rightarrow 3x-2y+z=-9$

13. B- $BA^2=\begin{bmatrix}2 & 4\\ 1 & 0\end{bmatrix}\begin{bmatrix}-5 & 15\\ -10 & 10\end{bmatrix}=\begin{bmatrix}-50 & 70\\ -5 & 15\end{bmatrix}$

14. A - $\begin{vmatrix}i & j & k\\ 1 & -2 & 1\\ 2 & 3 & -1\end{vmatrix}=2i+2j+3k-(-4k+3i-j)=-i+3j+7k$

15. C - $W=(50)(10)\cos 30^\circ = 250\sqrt{3}$

16. A - $A^{-1}=\begin{bmatrix}-3/7 & 3/7 & 2/7\\ -2/7 & 9/7 & 6/7\\ -1/7 & 1/7 & 3/7\end{bmatrix}$. Since we are looking for the element in row 3, column 2, that would be $1/7$.

17. A - $\begin{bmatrix}-3 & 4\\ -2 & 5\end{bmatrix}\begin{bmatrix}2\\ 7\end{bmatrix}=\begin{bmatrix}22\\ 31\end{bmatrix}=(22-31)^2=81$

18. B – just shift the line left 3 and down 2.

19. C - $\begin{bmatrix}\cos 45 & -\sin 45\\ \sin 45 & \cos 45\end{bmatrix}\begin{bmatrix}-\sqrt{2}\\ \sqrt{2}\end{bmatrix}=\begin{bmatrix}-2\\ 0\end{bmatrix}$

20. C – reduced row echelon form requires an identity matrix on the left and the solutions on the Right .

$$\left[\begin{array}{ccc|c}1 & 0 & 0 & 1\\ 0 & 1 & 0 & -2\\ 0 & 0 & 1 & 3\end{array}\right].$$

21. D – the trace is just the sum of the numbers along the main diagonal.

22. E - $\cos\theta=\frac{6}{6\sqrt{10}}=\frac{\sqrt{10}}{10}\rightarrow \sin\theta=\sqrt{1-\frac{1}{10}}=\frac{3}{\sqrt{10}}=\frac{3\sqrt{10}}{10}$

23. A - $\frac{1}{x}\begin{bmatrix}0 & 1\\ -1 & 2\end{bmatrix}\rightarrow \frac{2}{x}=\frac{2}{5}\rightarrow x=5$

24. B – just a simple linear combination.

25. A – the rank of the matrix is the order of the highest order non-zero determinant which can be formed by deleting rows and/or columns. Therefore this matrix has an order of 2.

26. B - $V = \frac{1}{6}\begin{vmatrix} 2 & 1 & -3 & 1 \\ 1 & 0 & 2 & 1 \\ 3 & -1 & 1 & 1 \end{vmatrix} = \frac{1}{6}(6) = 1$

27. B - $\begin{vmatrix} \lambda - 2 & 1 \\ 1 & \lambda - 2 \end{vmatrix} = 0 \rightarrow \lambda^2 - 4\lambda + 3 = 0 \rightarrow \lambda = 1, 3$

28. A – solving the determinant gives the sum rule for cosine.

29. D – matrices do not have the same dimensions.

30. A – a skew matrix has the property $A^T = -A$, the only one that satisfies this condition is a.

Tiebreaker answers

1.Since 2 times the first row is equal to the 4th row, the determinant is 0.

2. The trace is the sum of the elements on the main diagonal, which would be 9.

3. A plane has coefficients equal to the cross product of the two vectors. The cross product is $x - 2y + 3z$. Plugging in any of the three points gives you the constant, so the equation of the plane is $x - 2y + 3z = -4$.