

2009 Matrices and Vectors (Alpha)

1. Which of the following vectors is perpendicular to the vectors  $\langle -2, 3, 1 \rangle$  and  $\langle 4, 1, -3 \rangle$ ?

- A.  $\langle -10, 2, -14 \rangle$       b.  $\langle 3, 2, 4 \rangle$       c.  $\langle -9, -10, 12 \rangle$       d.  $\langle 5, 1, 7 \rangle$

2. Simplify. 
$$\begin{vmatrix} \sin \theta & -\sin \theta & 1 - \sin \theta \\ \sin \theta & 1 & \cos \theta \\ -1 & 1 & 1 \end{vmatrix}$$

- a.  $1 + \sin \theta$       b.  $1 - 2\sin^2 \theta - \sin 2\theta + \sin \theta$       c.  $\cos^2 \theta$       d.  $2\sin \theta - \sin \theta \cos \theta$

3. If a triangle with coordinates at  $(3, 2)$ ,  $(5, 7)$ , and  $(11, 2)$  is transformed by the matrix  $\begin{bmatrix} -2 & -1 \\ 4 & 3 \end{bmatrix}$  to a triangle with coordinates  $(-8, 18)$ ,  $(-17, 41)$ , and  $(-24, 50)$ , then the difference of area of the new triangle to the old triangle is:

- A. 20      b. 30      c. 40      d. 50

4. What is the angle between the vectors  $\langle 3, 7, -9 \rangle$  and  $\langle 11, 12, 13 \rangle$ ?

- A.  $60^\circ$       b.  $90^\circ$       c.  $120^\circ$       d.  $135^\circ$

5. Find the resultant of the two vectors of length 30 and 36 which form an angle of  $60^\circ$ .

- A.  $6\sqrt{91}$       b.  $18\sqrt{17}$       c.  $66\sqrt{3}$       d.  $56\sqrt{37}$

6. Find the sum of the squares of  $\begin{vmatrix} 3 & -1 & 1 \\ x & x & 6 \\ x & 1 & -2 \end{vmatrix} = 6$ .

- A. 217      b. 169      c. 194      d. 121

7. Let  $a = \langle 5, 12 \rangle$ ,  $b = \langle 3, -4 \rangle$ ,  $c = \langle -15, 8 \rangle$ . If  $d$  is equal to  $(a \bullet c)b$ , then what is the magnitude of  $d$ ?

- A. 105      b. 561      c. 819      d.  $-561$

8. Let  $M = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$ . Find the sum of the elements in  $M^4 + M^3 + M^2 - M + I + M^{-1}$ , where  $I$  is the  $2 \times 2$  identity matrix.

- A. 8      b. 9      c. 10      d. 18

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9. When solving a system of three variables  $x$ ,  $y$ , and  $z$ , kevin used cramer's rule and found the

following for  $x$ :  $\frac{\begin{vmatrix} -4 & 2 & -1 \\ 3 & 1 & 1 \\ 5 & 1 & 2 \end{vmatrix}}{\begin{vmatrix} 3 & 2 & -1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix}}$ . Find the value of  $z$ .

- A. -2                      b. 0                      c. 1                      d. 3

10. Let  $u = \langle 1, -1 \rangle$ ,  $v = \langle 3, -2 \rangle$ , and  $w = \langle 1, 2 \rangle$ . If  $w = Au + Bv$ , where  $A$  and  $B$  are scalars, find  $(A+B)^2$ .

- A. 13                      b. 17                      c. 21                      d. 25

11. If  $A$  is a  $2 \times 2$  matrix such that  $\det(A) = 8$ , then find the value of  $\det(A^{-1})$ .

- A. 8                      b. 1                      c.  $\frac{1}{8}$                       d.  $\frac{1}{64}$

12. Find the equation of the plane normal to  $v = 3i - 2j + k$  and containing the point.  $(-1, 4, 2)$

- a.  $3x + 3y + z = 7$       b.  $3x - 2y + z = 9$       c.  $3x - 2y + z = -9$       d.  $-x + 4y + 2z = 15$

13. If  $A = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 4 \\ 1 & 0 \end{bmatrix}$ , find  $BA^2$ .

- A.  $\begin{bmatrix} 14 & 20 \\ -8 & 0 \end{bmatrix}$       b.  $\begin{bmatrix} -50 & 70 \\ -5 & 15 \end{bmatrix}$       c.  $\begin{bmatrix} 58 & -66 \\ -3 & 31 \end{bmatrix}$       d.  $\begin{bmatrix} 25 & -12 \\ 0 & 64 \end{bmatrix}$

14. Find  $v \times w$  if  $v = i - 2j + k$  and  $w = 2i + 3j - k$ .

- A.  $-i + 3j + 7k$                       b.  $i - 3j - 7k$                       c.  $-5$                       d.  $2\sqrt{21}$

15. Mark pulls on a box with a rope at an angle of  $30^\circ$  with a force of 50 newtons. If the box is moved 10 meters, how much work did mark do?

- A. 500                      b.  $250\sqrt{2}$                       c.  $250\sqrt{3}$                       d. 250

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16. If  $A = \begin{bmatrix} -3 & 1 & 0 \\ 0 & 1 & -2 \\ -1 & 0 & 3 \end{bmatrix}$ , then what is the entry in  $A_{(3,2)}^{-1}$ ?

- A.  $\frac{1}{7}$                       b.  $-\frac{1}{7}$                       c.  $\frac{6}{7}$                       d.  $-\frac{6}{7}$

17. If  $\begin{bmatrix} -3 & 4 \\ -2 & 5 \end{bmatrix}$  transforms the vector  $\begin{bmatrix} -3 & 4 \\ -2 & 5 \end{bmatrix}$  to the vector  $\langle x, y \rangle$ , then find  $x^2 - 2xy + y^2$ .

- A. 81                      b. 64                      c. 144                      d. 289

18. Find the equation of the line which is the image of  $y = 2x - 3$  under a with vector  $\langle -3, 2 \rangle$ .

- A.  $y = x - 3$                       b.  $y = 2x + 5$                       c.  $y = 2x$                       d.  $y = -3x + 2$

19. Find the image under a rotation of  $45^\circ$  for  $\begin{bmatrix} -\sqrt{2} \\ \sqrt{2} \end{bmatrix}$ .

- A.  $\begin{bmatrix} \sqrt{2} \\ -\sqrt{2} \end{bmatrix}$                       b.  $\begin{bmatrix} 2 \\ -2 \end{bmatrix}$                       c.  $\begin{bmatrix} -2 \\ 0 \end{bmatrix}$                       d.  $\begin{bmatrix} -2\sqrt{2} \\ 2\sqrt{2} \end{bmatrix}$

20. Which of the following is the reduced row echelon form matrix to  $\left[ \begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 2 & -4 & 2 & 16 \\ 3 & 1 & -1 & -2 \end{array} \right] ?$

- A.  $\left[ \begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & -3 \end{array} \right]$                       b.  $\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array} \right]$                       c.  $\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array} \right]$                       d.  $\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 15 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & -3 \end{array} \right]$

21. What is the trace of  $\begin{bmatrix} -3 & -3 & 6 & 9 \\ -4 & 5 & 8 & -12 \\ 12 & 0 & 2 & 0 \\ 24 & 3 & 9 & -8 \end{bmatrix} ?$

- A. 18                      b. 16                      c. -1                      d. -4

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22. What is sine of the angle between  $\langle 4, -2 \rangle$  and  $\langle 3, 3 \rangle$ ?

- A.  $\frac{3\sqrt{10}}{20}$       b.  $\frac{3\sqrt{10}}{20}$       c.  $\frac{3\sqrt{5}}{20}$       d.  $\frac{3\sqrt{5}}{20}$

23. Let  $A = \begin{bmatrix} 2 & -1 \\ x & 0 \end{bmatrix}$  and  $B$  is the inverse of  $A$ . If  $B_{(2,2)}$  is  $\frac{2}{5}$ , find  $x$ .

- A. 5      b.  $\frac{3}{2}$       c.  $-\frac{5}{2}$       d.  $\frac{7}{3}$

24. If  $a = \langle 1, -2, 3 \rangle$ ,  $b = \langle 4, 3, -5 \rangle$ , and  $c = \langle -2, 1, 3 \rangle$ , find  $3a - 2b + c$ .

- A.  $\langle 1, 1, 1 \rangle$       b.  $\langle -7, -11, 22 \rangle$       c.  $\langle -7, -11, 22 \rangle$       d.  $\langle -5, -4, 5 \rangle$

25. What is the rank of  $\begin{bmatrix} 5 & -2 & 1 & 3 & 4 \\ 0 & -1 & 2 & 6 & -3 \end{bmatrix}$ ?

- A. 2      b. 3      c. 4      d. 5

26. Find the eigenvalue(s) of  $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ .

- A. 1      b. 1, 3      c. -1, -3      d. -3

27. Find the volume of the tetrahedron with vertices  $A(2, 1, -3)$ ,  $B(1, 0, 2)$ ,  $C(3, -1, 1)$ , and  $D(0, 1, 3)$ .

- A.  $\frac{1}{6}$       b. 1      c. 3      d. 6

28. When  $\begin{vmatrix} \cos A & \sin B \\ \sin A & \cos B \end{vmatrix}$  is simplified it is

- A.  $\cos(A+B)$       b.  $\cos(A-B)$       c.  $\cos(AB)$       d. 1

29. Find the difference of  $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} - \begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix}$ .

- A.  $\begin{bmatrix} 8 & 10 & 3 \\ 13 & 14 & 4 \end{bmatrix}$       b.  $\begin{bmatrix} -6 & -6 & -3 \\ -5 & -4 & -6 \end{bmatrix}$       c.  $\begin{bmatrix} -6 & -6 & 3 \\ -5 & -4 & 6 \end{bmatrix}$       d. Not possible

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30. Which of the following is skewed?

A.  $\begin{bmatrix} 0 & 8 \\ -8 & 0 \end{bmatrix}$

b.  $\begin{bmatrix} 0 & 8 \\ 8 & 0 \end{bmatrix}$

c.  $\begin{bmatrix} 8 & 0 \\ 8 & 0 \end{bmatrix}$

d.  $\begin{bmatrix} 0 & -8 \\ -8 & 0 \end{bmatrix}$

Tiebreakers:

1.  $\begin{vmatrix} 3 & 4 & -2 & 3 \\ 0 & 1 & 5 & 0 \\ 2 & 0 & 4 & 7 \\ 6 & 8 & -4 & 6 \end{vmatrix} =$

2. If all the entries in a  $3 \times 3$  matrix is 3, the what is the trace of the matrix?

3. What is the equation of the plane containing the points  $(1, -4, 7)$ ,  $(1, -4, 2)$ , and  $(-8, 3, 5)$ ?