1. **C** $\sum_{n=0}^{10} 4+3(n-1) = \frac{11}{2}(1+31) = 176.$

2. **B** The common difference, d is -3 and $a_1 = 8$ Thus $\sum_{n=1}^{10} a_n = \frac{10}{2}(2\cdot 8 + -3(10-1)) = -55.$

3. **B** $\sum_{n=1}^{\infty} \frac{4^n - 8}{5^n} = \sum_{n=1}^{\infty}\left(\frac{4}{5}\right)^n - \sum_{n=1}^{\infty} 8\left(\frac{1}{5}\right)^n = \frac{\frac{4}{5}}{1-\frac{4}{5}} - \frac{\frac{8}{5}}{1-\frac{1}{5}} = 2.$

4. **A** $\prod_{n=2}^{25} \log_{n+1}(n+2) = \log_3 4 \cdot \log_4 5 \cdot \cdots \cdot \log_{26} 27 = \frac{\ln 4}{\ln 3}\cdot\frac{\ln 5}{\ln 4}\cdot\cdots\cdot\frac{\ln 27}{\ln 26} = \frac{\ln 27}{\ln 3} = \log_3 27 = 3.$

5. **A** $\sum_{n=1}^{9} \ln\left(e\frac{n}{n+1}\right) = \sum_{n=1}^{9}\left[1 + \ln\left(\frac{n}{n+1}\right)\right] = 9 + \ln\frac{1}{2} + \ln\frac{2}{3} + \cdots + \ln\frac{9}{10} = 9 + \ln\left(\frac{1}{2}\cdot\frac{2}{3}\cdot\cdots\cdot\frac{9}{10}\right) =$ $9 + \ln\frac{1}{10} = 9 - \ln(10).$

6. **C** $\frac{a_1}{1-r} = 8$, so $a_1 = 8(1-r)$ and $7 = a_1 + a_1 r + a_1 r^2 = a_1(1+r+r^2)$. Substituting, $7 = 8(1-r)(1+r+r^2)$, and $\frac{7}{8} = 1 - r^3$. Thus, $r = \frac{1}{2}$. $a_1 = 8(1-\frac{1}{2}) = 4.$

7. **E** In order for the identity to be true, $\frac{a}{b} = \frac{1}{2}$. There are infinitely many positive real numbers a, b less than 7 which satisfy the identity.

8. **B** $\prod_{n=1}^{10} \exp\left(\frac{i\pi n}{3}\right) = \exp\left(\frac{i\pi(1+2+\cdots+10)}{3}\right) = \exp\left(\frac{i55\pi}{3}\right) = e^{i\pi/3} = \cos(\pi/3) + i\sin(\pi/3)$. Thus, $\theta = \frac{\pi}{3}$.

9. **B** $\log x^3 + \log\frac{x^4}{y^2} + \log\frac{x^5}{y^4} + \cdots + \log\frac{x^{25}}{y^{44}} = \log\frac{x^{3+\cdots+25}}{y^{2+\cdots+44}} = \log\frac{x^{322}}{y^{506}}$. Thus, $a + b = 322 + 506 = 828$.

10. **C** $\sum_{n=2}^{20}\frac{1}{n^2-n} = \sum_{n=2}^{20}\frac{1}{n-1} - \frac{1}{n} = \left(\frac{1}{1}-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) + \cdots + \left(\frac{1}{19}-\frac{1}{20}\right) = 1 - \frac{1}{20} = \frac{19}{20}.$

11. **D** $\det(A) = \begin{vmatrix} 2^n & -2^n \\ 2-n & n-1 \end{vmatrix} = 2^n$. Thus $\sum_{n=1}^{\infty}\frac{1}{\det(A)} = \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n = 1.$

12. **E** $a_4 = a_1 r^3 = -16$, and $a_1 = 2i$, so $r^3 = 8i$. The third roots of r are $-2i, \sqrt{3}+i, -\sqrt{3}+i$. Thus $(-2i)^2 + (\sqrt{3}+i)^2 + (-\sqrt{3}+i)^2 = 0.$

13. **D** $\sum_{x=-2}^{2}\left\lfloor\frac{x}{3}+0.8\right\rfloor = \left\lfloor\frac{-2}{3}+0.8\right\rfloor + \left\lfloor\frac{-1}{3}+0.8\right\rfloor + \left\lfloor\frac{0}{3}+0.8\right\rfloor + \left\lfloor\frac{1}{3}+0.8\right\rfloor + \left\lfloor\frac{2}{3}+0.8\right\rfloor = 0+0+0+1+1 = 2.$

14. **D** $\sum_{n=1}^{8} a_n = 16 = \frac{8}{2}(2(-5) + d(8-1))$. Thus, $d = 2$, and $\sum_{n=1}^{6}\frac{d^n}{18} = \frac{\frac{2}{18}(1-2^6)}{1-2} = 7.$

15. **B**

I. There are 38 terms in $\sum_{n=4}^{42} a_n$ **FALSE** There are 39 terms.

II. If $S = r + ra + ra^2 + \cdots$, and $|r| < 1$, then the infinite series converges. **FALSE** This is true as long as the common ratio, a, has the property that $|a| < 1$.

III. If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$. **TRUE**

16. **A** We must have that $-1 < \left((x+1)^2 - 4\right) < 1$, and we are looking for all positive x for which this is true. This inequality holds when $x \in (-1+\sqrt{3}, -1+\sqrt{5})$. The only interval given that contains these values is $(-\infty, 2)$.

17. **A** $a_3 = a_1r^2 = 10$, $a_7 = a_1r^6 = 160$, so $r^4 = \frac{a_1r^6}{a_1r^2} = \frac{160}{10} = 16$. Thus $r = \pm 2$, and $a_2 = -5$ when $r = -2$.

18. **B** $a_1 = S_1 = 2(1)^2 + 6(1) = 8$. Also, $a_2 = S_2 - S_1 = 20 - 8 = 12$. Thus, $d = a_2 - a_1 = 12 - 8 = 4$. We now know that $a_3 = 16$ and $a_{15} = 64$. Since $\frac{a_t}{a_3} = \frac{a_{15}}{a_t}$, we have that $a_t = 32$. This is the 7th term, since $32 = 8 + 4(7-1)$.

19. **E** Since the partial sums S_n alternate between 1 and 0, the infinite sum diverges.

20. **C** Note that $S_n = \begin{cases} 1 & n \text{ is even} \\ 0 & n \text{ is odd} \end{cases}$. Now, hand calculations show that $\sigma_1 = \frac{1}{1}, \sigma_2 = \frac{1+0}{2} = \frac{1}{2}, \sigma_3 = \frac{1+0+1}{3} = \frac{2}{3}, \sigma_4 = \frac{1+0+1+0}{4} = \frac{1}{2}, \sigma_5 = \frac{1+0+1+0+1}{5} = \frac{3}{5}$. This suggests that $\sigma_n = \begin{cases} \frac{1}{2} & n \text{ is even} \\ \frac{\frac{n+1}{2}}{n} & n \text{ is odd} \end{cases}$. Clearly, $\lim_{n\to\infty} \frac{\frac{n+1}{2}}{n} = \frac{1}{2}$, and as σ_n is already $\frac{1}{2}$ when n is even, $\lim_{n\to\infty} \sigma_n = \frac{1}{2}$.

21. **C** $\frac{5^{n-1}}{6^n} = \frac{1}{6}\left(\frac{5}{6}\right)^{n-1}$. Thus, $\sum_{n=1}^{\infty} \frac{5^{n-1}}{6^n} = \frac{\frac{1}{6}}{1-\frac{5}{6}} = 1$.

22. **E** In round one, we remove one segment of length $\frac{1}{3}$. In round two, we remove two segments, each of length $\frac{1}{9}$. In round three, we remove four segments, each of length $\frac{1}{27}$. Thus, in each round, we remove a total length of $\frac{1}{3}\left(\frac{2}{3}\right)^{n-1}$. Thus, since the procedure is carried on infinitely, we have $\sum_{n=1}^{\infty} \frac{1}{3}\left(\frac{2}{3}\right)^{n-1} = \frac{\frac{1}{3}}{1-\frac{2}{3}} = 1$.

23. **B** Note that $\frac{2}{5}$ was part of the first open segment removed. Clearly, $\frac{7}{27}$ and $\frac{1}{27}$ are in the Cantor Set, as they are endpoints of closed segments, and we know that these points are never removed as part of middle thirds. If you remove more and more middle thirds, you may come to the seemingly logical conclusion that the only points in the Cantor set are fractions with a power of 3 in the denominator. You might be surprised, then, to find that numbers such as $\frac{1}{4}$ and $\frac{3}{4}$ are also in the Cantor Set. As it turns out, any value between 0 and 1, inclusive, which can be written with only 0's and 2's in its base three decimal representation is in the Cantor set. Thus, $\frac{1}{4} = 0.\overline{02}_{three}$.

24. **C** $a_1 = S_1 = 3(1)^2 = 3$, $a_2 = S_2 - S_1 = 3(2)^2 - 3 = 9 = a_1 + d$.

25. **B** The trick is to see that $\sum_{k=0}^{\infty} \log_{2^{2^n}}(x) = \sum_{k=0}^{\infty} \frac{\log_2 x}{2^n} = 2\log_2 x = \log_2 x^2$. Now $\log_2 x^2 = x$, so $2^x = x^2$; the only positive x that satisfies this equation is 2.

26. **C** $\dfrac{A_6}{G_6} = \dfrac{\frac{6}{2}[2a_1 + 5(2)]}{a_1(2)^5} = \dfrac{1}{2}$, so $a_1 = 3$. Thus, we have $G_6 - A_6 = 96 - 48 = 48$.

27. **A** Let $x = \frac{1}{2}$. Then $1+4x^2+7x^4+10x^6+13x^8+\cdots = \dfrac{1 + 3x^2 + 3x^4 + 3x^6 + \cdots}{1 - x^2} = \dfrac{1 + 3(x^2 + x^4 + x^6 + \cdots)}{1 - x^2} = \dfrac{1 + 3\frac{x^2}{1-x^2}}{1 - x^2} = \dfrac{2x^2 + 1}{(1 - x^2)^2} = \dfrac{8}{3}$ when plugging in $x = \frac{1}{2}$ and simplifying.

28. **D** $A_{sector} = \frac{\theta}{2}r^2$, so since the radius is fixed, the angles also form an arithmetic progression, and $\theta_1 + \theta_2 + \cdots + \theta_{13} = 2\pi$, and if θ_1 is the smallest angle, and θ_{13} is the largest angle, then we have $\theta_{13} = 3\theta_1$. Thus, $\frac{13}{2}(\theta_1 + 3\theta_1) = 2\pi$, and $\theta_1 = \frac{\pi}{13}$, so the largest angle is then $\frac{3}{13}\pi$ and $p = 3, q = 13$.

29. **B** $\displaystyle\sum_{n=1}^{\infty}(1 - z)^n = \frac{1 - z}{1 - (1 - z)} = \frac{1 - z}{z} = z^{-1} - 1.$

30. **B** $\displaystyle\sum_{n=7}^{23} a_n = \frac{17}{2}(5 + 17) = 187$. Note that there are 17 terms.

$\boxed{\textbf{TB1}}$ $\boxed{\frac{2}{5}}$ Since $\deg(5n^3 + 3n^5 + 2n^7 + 7n^2) = \deg(3n^3 + 5n^2 + 7n^5 + 5n^7)$, the limit is the ratio of leading coefficients, $\frac{2}{5}$.

$\boxed{\textbf{TB2}}$ $\boxed{2091}$ The smallest multiple of 3 is 12, and the largest is 111. There are $\frac{111-12}{3} + 1 = 34$ multiples of 3, including 12 and 111, so the sum is $\frac{34}{2}(12 + 111) = 2091$.

$\boxed{\textbf{TB3}}$ $\boxed{7}$ The digits of the expansion of $\frac{1}{7} = 0.\overline{142857}$ repeat with frequency six. The remainder of 138 when divided by 6 is 0, so the 138th digit is 7.