A few friendly reminders: NOTA means None Of The Above, and we define the imaginary unit as $i = \sqrt{-1}$. Now take a deep breath, enjoy, and good luck!

1. Evaluate $\sum_{n=0}^{10} 4 + 3(n-1)$.

   A. 165 B. 169 C. 176 D. 190 E. NOTA

2. Let a_n be an arithmetic sequence, with $a_2 = 5$ and $a_9 = -16$. Find the sum of the first 10 terms.

   A. -55 B. -5 C. -115 D. 155 E. NOTA

3. Find $\sum_{n=1}^{\infty} \frac{4^n - 8}{5^n}$.

   A. -5 B. 2 C. 6 D. 15 E. NOTA

4. Evaluate $\prod_{n=2}^{25} \log_{n+1}(n+2)$.

   A. 3 B. $2\log(3)$ C. 9 D. 1 E. NOTA

5. Evaluate $\sum_{n=1}^{9} \ln\left(e\frac{n}{n+1}\right)$.

   A. $9 - \ln(10)$ B. $9e - \ln(9)$ C. 8 D. 10 E. NOTA

6. The sum of an infinite geometric sequence is 8, and the sum of the first 3 terms is 7. Find the first term.

   A. $\frac{1}{2}$ B. $3\sqrt{2}$ C. 4 D. 6 E. NOTA

7. Suppose $\sum_{n=0}^{\infty} \left(\frac{a}{b}\right)^n = 2$. How many positive, real coordinate pairs (a, b), with $a, b < 7$, satisfy the identity?

   A. 3 B. 4 C. 6 D. 7 E. NOTA

8. For the sake of notational clarity, mathematicians often write $e^x = \exp(x)$. Now consider $\prod_{n=1}^{10} \exp\left(\frac{i\pi n}{3}\right)$, which can be simplified as $cis(\theta)$. Find the smallest non-negative value of θ such that $\prod_{n=1}^{10} \exp\left(\frac{i\pi n}{3}\right) = cis(\theta)$.

   A. 0 B. $\frac{\pi}{3}$ C. $\frac{2\pi}{3}$ D. π E. NOTA

9. Consider the sum of the first 23 terms of the series $\log x^3 + \log\left(\frac{x^4}{y^2}\right) + \log\left(\frac{x^5}{y^4}\right) + \cdots$. This sum can be written in the form $\log\left(\frac{x^a}{y^b}\right)$. Find $a+b$.

A. 805 B. 828 C. 851 D. 854 E. NOTA

10. Find $\displaystyle\sum_{n=2}^{20} \frac{1}{n^2 - n}$.

A. $\frac{1}{20}$ B. $\frac{9}{20}$ C. $\frac{19}{20}$ D. 1 E. NOTA

11. Define the matrix $A = \begin{pmatrix} 2^n & -2^n \\ 2-n & n-1 \end{pmatrix}$. Find $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\det(A)}$.

A. -1 B. $-\frac{1}{3}$ C. $-\frac{1}{2}$ D. 1 E. NOTA

12. Let a_n be the n^{th} term of a complex-valued sequence, where $a_1 = 2i$ and $a_4 = -16$. Find the sum of the squares of all possible common ratios.

A. -8 B. $8i$ C. $i(2-2\sqrt{3})$ D. $i(4-\sqrt{3})$ E. NOTA

13. Find $\displaystyle\sum_{x=-2}^{2} \left\lfloor \frac{x}{3} + 0.8 \right\rfloor$, where $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

A. -1 B. 0 C. 1 D. 2 E. NOTA

14. Suppose $\displaystyle\sum_{n=1}^{8} a_n = 16$ with $a_1 = -5$. If a_n is arithmetic and d is the common difference, find $\displaystyle\sum_{n=1}^{6} \frac{d^n}{18}$.

A. 6 B. $-\dfrac{31}{9}$ C. $\dfrac{31}{9}$ D. 7 E. NOTA

15. Which of the following are true?

I. There are 38 terms in $\displaystyle\sum_{n=4}^{42} a_n$.

II. If $S = r + ra + ra^2 + \cdots$, and $|r| < 1$, then the infinite series converges.

III. If $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, then $\displaystyle\lim_{n\to\infty} a_n = 0$.

(A) II only (B) III only (C) II, III (D) I, II, III (E) NOTA

16. Consider $\sum_{n=42}^{\infty}\left((x+1)^2-4\right)^n$, where $x \in \mathbb{R}$. Select the smallest interval below which contains all positive values of x for which the series converges.

(A) $(-\infty, 2)$ (B) $(-2, 1]$ (C) $(0, 1.01)$ (D) $(1, \infty)$ (E) NOTA

17. If the 3rd term of a real geometric sequence is 10, and the 7th term is 160, which of the following is a possible value for the second term?

(A) -5 (B) $\frac{5}{2}$ (C) $-\frac{5}{2}$ (D) 8 (E) NOTA

18. Let $S_n = \sum_{k=1}^{n} a_n = 2n^2 + 6n$, where a_n is the nth term of an arithmetic sequence. the terms a_3, a_t and a_{15} comprise consecutive terms of a geometric sequence. Find t.

(A) 5 (B) 7 (C) 8 (D) 9 (E) NOTA

For the next two questions, let $u_k = (-1)^{k-1}$.

19. Find the following sum: $\sum_{k=1}^{\infty} u_k$.

(A) $\frac{1}{2}$ (B) -1 (C) 0 (D) 1 (E) NOTA

20. Now define $S_n = \sum_{k=1}^{n} u_k$ and $\sigma_n = \frac{1}{n}\sum_{k=1}^{n} S_k$. If $\lim_{n\to\infty} \sigma_n$ exists, we call the limit the Cesáro sum, and say u_k is Cesáro summable. Indicate the Cesáro sum, if it exists.

(A) 0 (B) 1 (C) $-\dfrac{1}{2}$ (D) $\dfrac{1}{2}$ (E) NOTA

21. Find the sum of the infinite sequence whose nth term is $\dfrac{5^{n-1}}{6^n}$. Assume the series starts when $n = 1$.

(A) $\dfrac{5}{6}$ (B) $\dfrac{6}{5}$ (C) 1 (D) 25 (E) NOTA

Use the following information for the next two questions: Let C represent the Cantor set, which is created as follows: start with the unit interval $[0, 1]$ and remove the open "middle third" interval, $(\frac{1}{3}, \frac{2}{3})$. From the two remaining closed segments, $[0, \frac{1}{3}]$ and $[\frac{2}{3}, 1]$, remove the two open middle third segments, $(\frac{1}{9}, \frac{2}{9})$ and $(\frac{7}{9}, \frac{8}{9})$. There will now be four closed segments remaining; remove the four open middle third segments from the four closed segments. Repeat this process ad infinitum. Note that infinitely many points in $[0, 1]$, including segment endpoints such as $0, 1, \frac{1}{3}$, etc. will never be removed by this process. These remaining points are called the Cantor set, C.

22. Find the total length of all removed middle third segments. You may assume that the length of an open interval (x, y) is the same as the closed interval $[x, y]$.

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{8}{9}$ (E) NOTA

23. Which of the following points is *not* a member of the Cantor Set, C?

I. $\frac{1}{27}$

II. $\frac{7}{27}$

III. $\frac{2}{5}$

(A) I only (B) III only (C) I, II (D) II, III (E) NOTA

24. The sum of the first n terms of an arithmetic sequence are given by the formula $S_n = 3n^2$. Find the sum of the first term and the common difference.

(A) 3 (B) 6 (C) 9 (D) 12 (E) NOTA

25. Find the sum of all positive real solutions of $\sum_{k=0}^{\infty} \log_{2^{2^n}}(x) = x$.

(A) 1 (B) 2 (C) 4 (D) $\log(2)$ (E) NOTA

26. Let A_6 be the sum of the first six terms of an arithmetic sequence, and G_6 be the sum of the first six terms of a geometric sequence. Both sequences share the same first term, and the common difference of the arithmetic sequence equals the common ratio of the geometric sequence, which equals two. Given that $\frac{A_6}{G_6} = \frac{1}{2}$, find $G_6 - A_6$.

(A) 1 (B) 3 (C) 48 (D) 144 (E) NOTA

27. Find the sum of the infinite series: $1 + 4\left(\frac{1}{2}\right)^2 + 7\left(\frac{1}{2}\right)^4 + 10\left(\frac{1}{2}\right)^6 + 13\left(\frac{1}{2}\right)^8 + \cdots$.

(A) $\frac{8}{3}$ (B) $e^{\frac{\pi}{2}}$ (C) $\frac{9}{2}$ (D) $\frac{11}{2}$. (E) NOTA

28. A circular disk is cut into thirteen sectors whose areas form an arithmetic sequence. The angle of the largest sector is three times the angle of the smallest sector. Let $\frac{p}{q}\pi$ be the size of the angle of the largest sector, where $\frac{p}{q}$ is in lowest terms. Which of the following represents the sum $p + q$?

(A) 2 (B) 12 (C) 13 (D) 16 (E) NOTA

29. Find $\sum_{n=1}^{\infty}(1-z)^n$. Assume the series converges.

(A) z^{-1} (B) $z^{-1} - 1$ (C) $\frac{z-1}{z-2}$ (D) 1 (E) NOTA

30. Find $\sum_{n=7}^{23} a_n$ when $a_7 = 5$ and $a_{23} = 17$, and a_n is arithmetic.

(A) 198 (B) 187 (C) 171 (D) 165 (E) NOTA

TB1 Let $a_n = \dfrac{5n^3 + 3n^5 + 2n^7 + 7n^2}{3n^3 + 5n^2 + 7n^5 + 5n^7}$. Find $\lim_{n \to \infty} a_n$.

TB2 Find the sum of all multiples of three between 11 and 111, inclusive.

TB3 Find the 138th digit after the decimal point in the expansion of $\frac{1}{7}$.