1. **B** $\frac{8^3-2^3}{3}+\frac{8^2-2^2}{2}+(8-2)+\ln(8)-\ln(2)=168+30+6+\ln(4)=204+2\ln(2)$.

2. **B** $f(x)=x^2+x+1+\dfrac{\frac{1}{x}}{1-\frac{1}{x}}=x^2+x+1+\dfrac{1}{x-1}$; $\dfrac{1}{4-2}\displaystyle\int_2^4 x^2+x+1+\frac{1}{x-1}dx=$ $\frac{1}{2}\left(\frac{56}{3}+6+2+\ln(3)\right)=\frac{40}{3}+\ln\sqrt{3}$.

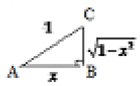
3. **A** Let r be the inner radius; then $A=\pi(13^2-r^2)\rightarrow\frac{dA}{dt}=-2\pi r\frac{dr}{dt}=-52\pi$.

4. **B** $1+\sin(2x)=(\sin^2(x)+\cos^2(x)+2\sin(x)\cos(x))=(\sin(x)+\cos(x))^2$. Letting $u=\sin(x)+\cos(x)$, this is $\displaystyle\int\frac{du}{u^2}=-\frac{1}{u}=-\frac{1}{\cos(x)+\sin(x)}$. Plugging in, we get $2-\sqrt{3}$.

5. **C** We have $4a^2+b^2=9$; this is an ellipse, with semimajor axis 3 and semiminor axis $\frac{3}{2}$; area is $\frac{9\pi}{2}$.

6. **C** The trick here is not subtracting 16 from both sides. If $f(x)<g(x)$, then $x^4-4x^3+6x^2-4x+1<16$; thus, $(x-1)^4<16$, and $-1<x<3$. The area of the region is thus $\displaystyle\int_{-1}^{3}16-(x-1)^4dx=\frac{256}{5}$.

7. **C** $2\pi\displaystyle\int_0^\infty xe^{-x^2}dx=-\pi\int_0^\infty -2xe^{-x^2}dx=\pi$.

8. **B** Consider the diagram 

. Note that $\arccos(x)+\arcsin(x)=\text{m}\angle\text{A}+\text{m}\angle\text{C}=\frac{\pi}{2}$, regardless of x; hence, the area bound by the line $y=\frac{\pi}{2}$ on $(0,\frac{1}{2})$ is $\frac{\pi}{4}$.

For questions 9-11, rewrite the equation of the circle as $(x-2)^2+(y-1)^2=25$.

9. **B** The given line passes through the center of the circle; the solid formed is just a sphere of radius 5, with volume $\frac{4\pi}{3}(5^3)=\frac{500\pi}{3}$.

10. **D** Making the substitutions $u=x-2$ and $t=y-1$, this is the same thing as asking for the area of the region bound by $u^2+t^2=25, t=0$, and $t=2.5$, or $2\displaystyle\int_0^{2.5}\sqrt{25-t^2}dt=\frac{25\pi}{6}+\frac{25\sqrt{3}}{4}$. Note the 2 outside the integral, because the integral itself adds the limitation $u>0$, which is not present.

11. **B** Making the same substitutions as in question 12, we need to find an integral which will give the volume of the solid formed when the part of $u^2+t^2=25$ above $t=1$ is rotated about $t=1$; we can do this by using the shell method, and getting $4\pi\displaystyle\int_1^5(t-1)\sqrt{25-t^2}dt$ (it's 4π rather than 2π for the same reason as in question 12).

12. **C** $A=2k(4-k^2)=8k-2k^3\rightarrow\frac{dA}{dk}=8-6k^2\rightarrow k=\frac{2\sqrt{3}}{3}$

13. **C** Let k represent the distance traveled since Hurricane Zeta started pushing; then, $f(x)=-x^2+2x+k$. The roots of this parabola are $\frac{-2\pm\sqrt{4+4k}}{-2}=1\pm\sqrt{1+k}$, and the area will be $\displaystyle\int_{1-\sqrt{k+1}}^{1+\sqrt{k+1}}-x^2+2x+kdx$. Note that we can't just differentiate this straight up, since there's a k in the integrand. We split this integral into $\displaystyle\int_{1-\sqrt{k+1}}^{1+\sqrt{k+1}}-x^2+2xdx$ and $\displaystyle\int_{1-\sqrt{k+1}}^{1+\sqrt{k+1}}kdx$. The first can be done by ordinary methods, and for the second we'll integrate, set in terms of k, then differentiate. After doing this (the process is simple enough), we obtain $\frac{dA}{dt}=2\sqrt{k+1}\frac{dk}{dt}$; $\frac{dk}{dt}=2$, so $2(3)(2)=12$.

14. **C** The centroid of this triangle is the average of the coordinates, or (3, 1). Using the determinants method, the area of this triangle is $\frac{9}{2}$. Since $V=2\pi rA$ by the Theorem of Pappus, we need the vertex that's the furthest from the centroid; this is (6, 1) and hence $r=3\rightarrow A=27\pi$.

15. **C** $V=\pi\displaystyle\int_0^{\sqrt{k}}(k-x^2)dx=\frac{2\pi(k\sqrt{k})}{3}$. $D=\frac{M}{V}\rightarrow\frac{dD}{dt}=\frac{-M}{V^2}\frac{dV}{dt}$; plugging in, we get $-\frac{5}{54}$

16. **D** We need $\displaystyle\int_0^k x^3dx=\int_k^4 x^3dx\rightarrow k^4/4=64-k^4/4\rightarrow k^4=128\rightarrow k^2=8\sqrt{2}$.

17. **D** $|2|+|-2|=4$.

18. **B** $V=\displaystyle\int_0^2\pi r^2dx=\int_0^2\pi(\frac{3^x}{2})^2dx=\frac{\pi}{4}\int_0^2 9^xdx=\frac{10\pi}{\ln(3)}$.

19. **D** Writing out a few terms, we see that this will be equivalent to $1(1) + \frac{1}{2}\left(\frac{3}{2}\right)^2 + \frac{1}{4}\left(\frac{7}{4}\right)^2 + \frac{1}{8}\left(\frac{15}{8}\right)^2 + ... = 1 + \frac{9}{8} + \frac{49}{64} + \frac{225}{512} + ... = \sum_{k=1}^{\infty}\left(\frac{(2^k-1)^2}{2^{3k-3}} = 8\sum_{k=1}^{\infty}\frac{1}{2^k} + \frac{1}{8^k} - \frac{2}{4^k}\right) = \frac{80}{21}$

20. **E** $\int_0^2 x^2 dx = \frac{8}{3}$.

21. **E** The radius of the cylinder formed will be n and the height will be m. Since $2m + 2n = 24$, $m + n = 12 \rightarrow m = 12 - n$, and thus $V = \pi n^2(12 - n)$. n can take on values between 0 and 12; hence, the integral is $\frac{1}{12}(\pi)\int_0^{12}(12n^2 - n^3)dn = \frac{1}{12}(\pi)(4n^3 - n^4/4) = \frac{\pi}{12}(12^3)(4-3) = 144\pi$.

22. **C** $\text{SA} = 2\pi\int_0^1 \frac{4}{3}x^3\sqrt{1+(4x^2)^2}dx = 2\pi\int_0^1 \frac{4}{3}x^3\sqrt{1+16x^4}dx = \frac{\pi}{36}(17\sqrt{17}-1)$

23. **B** $\pi\int_0^{16}(\sqrt[4]{y})^2 dy = \frac{16\pi\sqrt[4]{5}}{2}$

24. **C** $\pi\int_0^2\left(16^2 - \left(x^4\right)^2\right)dx = \frac{4096\pi}{9}$

25. **D** I and IV will always overestimate the integral; II will always underestimate; III is impossible to determine without knowing the actual function.

26. **B** $(0^2+2) + (1^2+2) + (2^2+2) + (3^2+2) = 22$.

27. **B** $2\pi\int_0^a x(ax^2)dx = \pi\int_0^a a^2x^4 dx \rightarrow \frac{a^5}{2} = \frac{a^7}{5} \rightarrow a = \sqrt{\frac{5}{2}}$; smallest integer above this is 2.

28. **B** The area under the curve, when rotated, would be $2\pi\int_0^{\frac{\pi}{2}} x\sin(x)dx$; since we're concerned with the area above the curve and below $y = 1$, however, this will be $\pi\left(\frac{\pi}{2}\right)^2 - \int_0^{\frac{\pi}{2}} x\sin(x)dx = \frac{\pi^3}{4} - 2\pi$

29. **C** $\pi\int_0^{\frac{\pi}{2}} 1^2 - \sin^2(x)dx = \frac{\pi^2}{4}$

30. **E** Using the determinants method, $A = \frac{a^2+b^2-4}{2} \rightarrow \frac{dA}{dt} = a\frac{da}{dt} + b\frac{db}{dt} \rightarrow 10(2) + 14(3) = 62$