1. C For the first hour, the snowmobile travels at 20mph. Hence, at 3PM, there are 265 miles left. Now, since each event has equal probability, the expected value \( E(v) \) of the snowmobile’s velocity is \( 20 + \frac{(4k-6+k^2)}{2} \). We want to maximize \( \frac{dv}{dt} \); hence, \( k = 2 \), \( E(v) = 19 \); 265/19 will give 13\(\frac{18}{19} \) more hours, which is close to 5:00 AM.

2. C The “appropriate vertical lines” are \( x = 1 \) and \( x = 10 \); hence, for some curve \( f(x) \), we need \( \int_1^{10} f(x)dx = 1 \) (since the area under a probability density function is 1). Only choice C satisfies this.

3. A This is just \( \frac{1}{2} \times \frac{1}{3} = \frac{1}{15} \). 4. D \( \frac{dv}{dt} = \frac{dv}{dt} \rightarrow ln(v) = k ln(t + 1) + C \rightarrow v = C(t + 1)^k; v(0) = 200 \rightarrow C = 200; v(3) = 100 \rightarrow k = -\frac{1}{2}; 200(8)^{-\frac{1}{2}} = 50\sqrt{2}.

5. D Let \( x \) represent the distance of the first snowmobile, \( y \) the second, and \( z \) the distance between them; after two hours, \( x = 200 \) and \( y = 40 \), so \( z = \sqrt{40^2 + 200^2} = 40\sqrt{1 + 25} = 40\sqrt{26} \). Since \( x^2 + y^2 = z^2 \), \( x \frac{dx}{dt} + y \frac{dy}{dt} = z \frac{dz}{dt} = 200(50) + 40(20) = 40\sqrt{26} \frac{dz}{dt} \rightarrow \frac{5(50)+20}{\sqrt{26}} = \frac{135\sqrt{26}}{13}.

6. B \( v(t) = \left( \frac{1}{t+1}, 6t, 0 \right) \) \( \rightarrow \) speed at \( t = 1 \) is \( \sqrt{\frac{1}{4} + 36} = \frac{\sqrt{145}}{2} \).

7. D From the right, \( \frac{x}{|x|} \) goes to \( \infty \) and \( \frac{|x|}{x} \) goes to 0, whereas from the left it’s vice versa; limit is \( \infty \).

8. C \( \frac{dN}{dt} = N + 2009 \rightarrow ln(N + 2009) = t + C \rightarrow N = Ce^t - 2009; N(0) = 1337 \rightarrow C = 3346 \). \( N(10) = 33460 - 2009 = 31451 \).

9. B To find the bounds of the LPE and given Lorenz curve, set \( P = Y(P) \rightarrow P = 2P^3 - P^3 \rightarrow P(P - 1)^2 = 0; P = 0, 1 \). We want \( \frac{1}{\int_0^1 PdP - \int_0^1 (2P^2 - P^3)dP} = \frac{1}{\int_0^1 PdP} = \frac{1}{6} \).

10. D I–not necessarily true; the poorest portion of the population could, in theory, own zero wealth. II–true, if \( L \) is constant or linear over an interval then it is neither concave up nor concave down. III–true, IV–true, it makes no sense for the poorest portion of the population to own more than \( k\% \) of the wealth. V–false. Suppose that, for example, the poorest 50% of the population own 40% of the wealth; then, the poorest 55% cannot own less than 44%, because the extra 5% is richer than the poorest 50.

11. SA = 4\(\pi r^2 \rightarrow \frac{dA}{dt} = 8\pi r \frac{dr}{dt} \rightarrow \frac{dA}{dt} = 8\pi \times 7 \times 2 = 112\pi \).

12. B Differentiate to get \( -3x^2 + 4x = 0 \rightarrow x(-3x + 4) = 0 \rightarrow x = 0, 40 \frac{40}{4} \rightarrow 40 \frac{3}{4} \).

13. The intersection will be at \( x = \pi \). Hence \( \int_0^\pi (\sin(x) - x - \pi)dx = \frac{\pi^2}{2} + 2 \).

14. B \( \frac{dB}{dt} = \frac{1}{t+1} \rightarrow B = \ln(t + 1); L = t^2 + 2t + 1 \rightarrow dL = 2(t + 1)dt; 2\int_0^2 (t + 1)\ln(t + 1)dt = 2\int_1^3 u\ln(u)du \). Use integration by parts to obtain \( 9\ln(3) - 4 \), and divide by \( \mu_0 \).

15. A \( \int BbdL = \xi \rightarrow \xi = 100(t^3 + t^2) + C; \frac{dc}{dt} = 100(3t^2 + 2t) \). Using a step size of 0.1, we have \( 1314 + (0.1)(100)(3t^2 + 2t) = 1314 + 10(0.23) = 1316.3 \).

16. B \( \int_1^7 x^2 dx = 19 \).

17. E Choice A fails when one of \( \{a_1, a_2, ..., a_k\} \) is -1 because of the \( \ln(x) \) obtained; choice B fails because there will be constants in front of the powers of \( x \); C fails when one of \( \{a_1, a_2, ..., a_k\} \) is negative; D fails when all of \( \{a_1, a_2, ..., a_k\} \) is positive. Hence, none of these statements are true.

18. A Any two adjacent sides sum to 10; consider one side to be \( x \) and the other to be \( 10 - x \); \( A = x(10 - x) \rightarrow \) the average value of these rectangles is \( \frac{1}{10} \int_0^{10} (10x - x^2)dx = \frac{50}{3} \).
19. D  \( A = k^2 \to dA = 4 = 2dk \). Since the cube maintains its shape, the height must also be increasing; \( V = k^3 \to dV = 3k^2dk = \frac{3k}{2} (2dk) = 6k \).
20. A  Setting \( \sin(x) = 2 \cos(x) \) and using \( \sin^2(x) + \cos^2(x) = 1 \) yields \( \sin(x) = -\frac{2}{\sqrt{5}} \) and \( \cos(x) = -\frac{1}{\sqrt{5}} \). \( r'(x) = 2 \cos(2x) + \sec^2(x) = 2(2 \cos^2(x) - 1) + 5 = \frac{19}{5} \); multiplying by \( \cos(x) \) gives \( -\frac{19\sqrt{5}}{25} \).
21. B  \( \int_0^4 k\sqrt{4-x}dx = \frac{-2k}{3}(4-x)^{3/2} \) from 0 to 4; this is \( \frac{16k}{3} \to k = \frac{3}{16} \).
22. E  It can be shown that \( f(x) \) is its own inverse; \( f(x) = g(x) \to f'(x) = g'(x) \to f'(x) = g(x) = 1 \) for all \( x \).
23. C  The inverse of \( f(x) \) can be shown to be \( \frac{dx+3}{ex-a}; g'(x) = \frac{dx-da-dex-bc}{(ex-a)^2} = \frac{-da-bc}{(ex-a)^2} \); to solve the given equation, \( da + bc = (cx - a)^2 \to c^2x^2 - 2acx + a^2 - da - bc = 0 \to \) product of roots is \( \frac{a^2-ad-bc}{c^2} \).
24. A  \( V = \pi \int_0^1 (\sqrt{x})^2 - (x^2)^2 dx = \pi \int_0^1 x - x^4 dx = \frac{3\pi}{10} \).
25. D  \( 11 + \frac{1}{2\sqrt{121}} \times (132 - 121) = 11 + \frac{11}{22} = \frac{23}{2} \).
26. C  Let the first number generated be \( x \) and the second be \( y \); then, \( xy < e^3, x < e^2, y < e^2 \) is the region in which I pay you. The area of this region is \( \int_e^{e^2} \frac{3}{x}dx + e(e^2) = 2e^3 \); hence, the area of the region in which you pay me is \( e^4 - 2e^3 \). My expected winnings/losses are given by \( \frac{(e^4 - 2e^3)(2k) - (2e^3)(2k^2 + 1)}{e^4} \); taking the derivative of the top gives \( 2(e^4 - 2e^3) - 2e^3(4k) \). Dividing through by \( 2e^3 \) and setting this equal to 0 gives \( k = \frac{e^2}{4} \).
27. E  \( \lim_{n \to \infty} \sum_{k=1}^{n} \left( \frac{2}{n} \left( \frac{2k^2}{n^2} - 3 \right)^2 \right) \) would give a definite integral; because of the extra \( k \) in the summand, however, this series is divergent.
28. B  \( f''(x) = 12x^2 + 12x - 144 = 12(x - 3)(x + 4) \); testing regions gives \((-4, 3)\).
29. C  \( x^2 - 2xy + y^2 = 16 \to (x - y)^2 = 16 \to x - y = 4 \to y = x - 4 \); \( \frac{dy}{dx} = 1 \) for all \( x \).
30. E  \( \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \ldots = 1 + (-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \ldots) = 1 - (1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots) = 1 - \ln 2 \).