

1. **B** $\frac{8^3-2^3}{3} + \frac{8^2-2^2}{2} + (8-2) + \ln(8) - \ln(2) = 168 + 30 + 6 + \ln(4) = 204 + 2\ln(2).$

2. **B** $(0^2+2) + (1^2+2) + (2^2+2) + (3^2+2) = 22.$

3. **A** It's just a quarter-circle; hence, $\frac{1}{4}(2\pi)(4) = 2\pi.$

4. **E** $g(x) = \cos(x) - i \sin(x); \int_0^\pi \cos(x) - i \sin(x) dx = \sin(\pi) - \sin(0) + i \cos(\pi) - i \cos(0) = -2i.$

5. **C** $u = -x^2; du = -2x dx \rightarrow -\frac{1}{2} \int_0^1 -2xe^{-x^2} dx = -\frac{1}{2} (e^{-1} - e^0) = \frac{e-1}{2e}.$

6. **D** First, evaluate $f(x)$ using integration by parts; we see that $\int \sin(\ln(x)) dx = x \sin(\ln(x)) - \int \cos(\ln(x)) dx.$ Since this cancels with $g(x)$, we're left with $x \sin(\ln(x))$ from 0 to $e^{\frac{\pi}{2}}.$

This is $e^{\frac{\pi}{2}};$ while $\ln(0)$ would cause a problem by itself, $x \sin(\ln(x))$ goes to 0 as x goes to 0.

7. **E** $f'(x) + g'(x) = e^x(\sin(x) + \cos(x)) = e^{\frac{\pi}{2}}.$

8. **D** We need $\int_0^k x^3 dx = \int_k^4 x^3 dx \rightarrow k^4/4 = 64 - k^4/4 \rightarrow k^4 = 128 \rightarrow k^2 = 8\sqrt{2}.$

9. **D** $f(x) < 0$ on $(-1, 3);$ hence, $\int_{-5}^{-1} 3x^2 - 6x - 9 dx - \int_{-1}^3 3x^2 - 6x - 9 dx + \int_3^5 3x^2 - 6x - 9 dx = 224.$

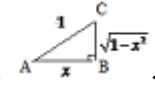
10. **C** The trick here is not subtracting 16 from both sides. If $f(x) < g(x)$, then $x^4 - 4x^3 + 6x^2 - 4x + 1 < 16;$ thus, $(x-1)^4 < 16,$ and $-1 < x < 3.$ The area of the region is thus

$$\int_{-1}^3 16 - (x-1)^4 dx = \frac{256}{5}.$$

11. **B** Break $\frac{6}{x^2-9}$ into partial fractions to obtain $\frac{1}{x-3} - \frac{1}{x+3};$ integrate to obtain $\ln(x-3) - \ln(x+3);$ plug in the limits to obtain $\ln 7 - \ln 3.$ 12. **D** $|2| + |-2| = 4.$

13. **B** Note that the odd powered terms cancel and so we have

120 $\int_{-1}^1 x^4 + x^2 dx = 2(120/5 + 120/3) = 128.$ 14. **D** I and IV will always overestimate the integral; II will always underestimate; III is impossible to determine without knowing the actual function.

15. **B** Consider the diagram . Note that $\arccos(x) + \arcsin(x) = m\angle A + m\angle C = \frac{\pi}{2},$ regardless of $x;$ hence $\int_0^{\frac{1}{2}} \frac{\pi}{2} dx = \frac{\pi}{4}.$

16. **A** Once again using a triangle, we see that $\sin(\arccos \frac{x}{2}) = \sqrt{1 - \frac{x^2}{4}} = \frac{1}{2}\sqrt{4 - x^2}.$ Using trig substitution to evaluate the integral, we get $\arcsin \frac{x}{2} + \frac{1}{2}\sqrt{1 - \frac{x^2}{4}};$ plugging in, we get $\frac{3\sqrt{3}+2\pi}{12}.$

17. **B** $1 + \sin(2x) = (\sin^2(x) + \cos^2(x) + 2\sin(x)\cos(x)) = (\sin(x) + \cos(x))^2.$ Letting $u = \sin(x) + \cos(x),$ this is $\int \frac{du}{u^2} = -\frac{1}{u} = -\frac{1}{\cos(x) + \sin(x)}.$ Plugging in, we get $2 - \sqrt{3}.$

18. **E** Clearly, a and r must be the roots of the polynomial, with $a < r;$ this would integrate the entire region on which the polynomial is positive. We find these roots to be $-\frac{5}{2}$ and 6; $[36 - 6.25] = 29.$

19. **E** It can be shown that $f(x)$ is its own inverse; hence, $f(x) - g(x) = 0$ and this integral is zero.

20. **C** $\int_{-\ln 2}^{\ln 2} \cosh(x) dx = 2 \int_0^{\ln 2} \cosh(x) dx = 2 \sinh(\ln 2) = 2 - \frac{1}{2} = \frac{3}{2}.$

21. **C** Let k represent the distance traveled since Hurricane Zeta started pushing; then,

$f(x) = -x^2 + 2x + k.$ The roots of this parabola are $\frac{-2 \pm \sqrt{4+4k}}{-2} = 1 \pm \sqrt{1+k},$ and the area will be

$\int_{1-\sqrt{k+1}}^{1+\sqrt{k+1}} -x^2 + 2x + k dx.$ Note that we can't just differentiate this straight up, since there's a k in the

integrand. We split this integral into $\int_{1-\sqrt{k+1}}^{1+\sqrt{k+1}} -x^2 + 2x dx$ and $\int_{1-\sqrt{k+1}}^{1+\sqrt{k+1}} k dx.$ The first can be done by ordinary methods, and for the second we'll integrate, set in terms of $k,$ then differentiate. After doing this (the process is simple enough), we obtain $\frac{dA}{dk} = 2\sqrt{k+1} \frac{dk}{dk}; \frac{dk}{dk} = 2,$ so $2(3)(2) = 12.$

22. **C** $f'(x) = 2x \sin(\sqrt{x^2}) \rightarrow f'(\frac{\pi}{3}) = \frac{2\pi}{3} \frac{\sqrt{3}}{2} = \frac{\pi\sqrt{3}}{3}$

23. **B** $\sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{(1-x)^2}{1-x^2}} = \frac{1-x}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}}$; integrating, we get $\arcsin(x) + \sqrt{1-x^2}$; plugging in, this is $\frac{\pi+3\sqrt{3}-6}{6}$.

24. **D** Let $u = \frac{x}{2}$; $dx = 2du$. Rewriting the integrand in terms of u , we get $2u^{2u}(1 + \ln(u)) = 2u^u(u^u(1 + \ln(u)))$. Since $u^u(1 + \ln(u))$ is the derivative of u^u , when integrating we get $(u^u)^2$; plugging in $u = 1$ and $u = 2$ (taken from the corresponding x -values), we get 15.

25. **A** $f(x) = x^3 - 6x^2 + 5x + C$; $f(0) = f(1) = C$; $f(2) = -6 + C \rightarrow 2C = -6 + C \rightarrow C = -6$; $f(-2) = -8 - 24 - 10 - 6 = -48$.

26. **B** Let $u = \cos(x) + 1 \rightarrow du = -\sin(x)$; this integral becomes $-\int_2^{\frac{3}{2}} \frac{du}{u} = -\ln \frac{3}{4} = \ln \frac{4}{3}$.

27. **A** Just graph the function on the interval.

28. **B** $kN = \frac{dN}{dt} \times t \rightarrow \frac{dN}{N} = k \frac{dt}{t} \rightarrow \ln(N) = k \ln(t) + C \rightarrow N = Ct^k$; plugging in $t = 1$ gives $C = 1337$ and plugging in $t = 2$ gives $k = 2$; hence, when $t = 3$, $N = 1337(3^2) = 1337(9) = 12033$.

29. **A** Let $x = \tan(u)$; the integrand becomes $\frac{\ln(\tan(u))}{(\tan^2(u)+1)}(\sec^2(u))du = \ln(\tan(u))du$, and the limits become 0 and $\frac{\pi}{2}$. Rewrite this integral as $\int_0^{\frac{\pi}{2}} \ln(\sin(u))du - \int_0^{\frac{\pi}{2}} \ln(\cos(u))du$; let $I_1 = \int_0^{\frac{\pi}{2}} \ln(\sin(u))du$ and let $I_2 = \int_0^{\frac{\pi}{2}} \ln(\cos(u))du$. Leaving I_1 alone, substitute $u = \frac{\pi}{2} - t$ for I_2 ; we get

$$\int_{\frac{\pi}{2}}^0 \ln(\cos(\frac{\pi}{2} - t))(-dt) = -\int_{\frac{\pi}{2}}^0 \ln(\sin(t))dt. \text{ Switching the bounds to get rid of the negative sign, this}$$

integral becomes $\int_0^{\frac{\pi}{2}} \ln(\sin(t))dt$; hence, $I_1 = I_2$, and the answer is zero. NOTE: This question should not be changed to E. Both I_1 and I_2 , despite being improper integrals, are convergent (to a value of $\frac{-\pi \ln 2}{2}$)

30. **E** Note that the integral is dy , not dx ! It's just $-8x$.