The acronym NOTA denotes that "None of These Answers" are correct. DNE stands for "Does Not Exist". The domain and range of functions are assumed to be either the real numbers or an appropriate subset unless otherwise stated. $[x]$ represents the greatest integer $\leq x$. Good luck and have fun!

1. Evaluate $\int_2^8 \left(x^2 + x + 1 + \frac{1}{x}\right) dx$.

A. $198 + 2\ln 2$ B. $204 + 2\ln 2$ C. $210 + 2\ln 2$ D. $214 + 2\ln 2$ E. NOTA

2. Use the left rectangle approximation method with four equal subdivisions to approximate the area bounded by the curve $y = x^2 + 2$ and the x-axis between $x = 0$ and $x = 4$.

A. 14 B. 22 C. 29 D. 38 E. NOTA

3. Find the length of the curve $y = \sqrt{16 - x^2}$ on the interval (0, 4).

A. 2π B. 4π C. 6π D. 8π E. NOTA

4. The imaginary number $i = \sqrt{-1}$ may be treated as a constant under the integration sign. Let $f(x) = \text{cis}(x)$ (where $\text{cis}(x) = \cos(x) + i\sin(x)$), and let $g(x)$ be the complex conjugate of $f(x)$. Find $\int_0^\pi g(x)dx$.

A. 0 B. 2 C. $2i$ D. $1 - i$ E. NOTA

5. Evaluate: $\int_0^1 xe^{-x^2} dx$.

A. $\frac{e-1}{2}$ B. $\frac{2e-1}{2}$ C. $\frac{e-1}{2e}$ D. $\frac{2e-1}{2e}$ E. NOTA

For questions 6-7, let $f(x) = \int_0^{e^x} \sin(\ln t)dt$ and let $g(x) = \int_0^{e^x} \cos(\ln t)dt$.

6. Find $f\left(\frac{\pi}{2}\right) + g\left(\frac{\pi}{2}\right)$.

A. $-\frac{e^{\frac{\pi}{2}}}{2}$ B. 0 C. $\frac{e^{\frac{\pi}{2}}}{2}$ D. $e^{\frac{\pi}{2}}$ E. NOTA

7. Find $f'\left(\frac{\pi}{2}\right) + g'\left(\frac{\pi}{2}\right)$.

A. $-\frac{e^{\frac{\pi}{2}}}{2}$ B. 0 C. 1 D. $\frac{e^{\frac{\pi}{2}}}{2}$ E. NOTA

8. Consider the region bound by $f(x) = x^3$, the x-axis, $x = 0$, and $x = 4$. If the line $x = k$ divides this region into two pieces of equal area, what is k^2?

A. 4 B. $4\sqrt{2}$ C. 8 D. $8\sqrt{2}$ E. NOTA

9. Find the area bound by the x-axis and the curve $f(x) = 3x^2 - 6x - 9$ from $x = -5$ to $x = 5$.

A. 128 B. 160 C. 192 D. 224 E. NOTA

10. Let $f(x) = x^4 + 6x^2 + 1$ and let $g(x) = 4x^3 + 4x + 16$. Find the area of the region bound above by $g(x)$ and below by $f(x)$.

A. $\frac{64}{5}$ B. $\frac{128}{5}$ C. $\frac{256}{5}$ D. $\frac{512}{5}$ E. NOTA

11. Compute $\int_4^6 \frac{6dx}{x^2 - 9}$.

A. $\ln 7 - 2\ln 2$ B. $\ln 7 - \ln 3$ C. $\ln 2$ D. $3\ln 2 - \ln 3$ E. NOTA

12. Suppose f is a continuous function and $\int_{-2}^{2} f(x)dx = 0$ and $\int_0^2 f(x)dx = 2$. What is the area bound by $f(x)$, the x-axis, and the lines $x = -2$ and $x = 2$?

A. 0 B. 1 C. 2 D. 4 E. NOTA

13. Evaluate: $120\int_{-1}^{1} (x^5 + x^4 + x^3 + x^2 + x)dx$.

A. 64 B. 128 C. 192 D. 220 E. NOTA

14. Suppose that on the interval (0, 10), the function $f(x)$ is decreasing, concave up, and greater than zero. Which of the following methods, with 5 equal subdivisions, overestimates the value of $\int_0^{10} f(x)dx$ regardless of what $f(x)$ is?

I. Left Rectangular
II. Right Rectangular
III. Midpoint
IV. Trapezoid

A. I, III, IV only B. II, IV only C. I, III only D. I, IV only E. NOTA

15. Evaluate: $\int_0^{\frac{1}{2}} (\arcsin(x) + \arccos(x))dx$.

A. $\frac{\pi}{12} + 2 - \sqrt{3}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{6} + 2 - \sqrt{3}$ D. $\frac{\pi}{2}$ E. NOTA

16. What is the value of $\int_0^1 \sin\left(\arccos\left(\frac{x}{2}\right)\right) dx$?

A. $\frac{3\sqrt{3}+2\pi}{12}$ B. $\frac{3\sqrt{3}+4\pi}{12}$ C. $\frac{3\sqrt{3}+\pi}{6}$ D. $2\sqrt{3} + \pi$ E. NOTA

17. Evaluate: $\int_0^{\frac{\pi}{6}} \frac{\cos(x)-\sin(x)}{1+\sin(2x)}dx$.

A. $\sqrt{3}-2$ B. $2-\sqrt{3}$ C. $\sqrt{3}-1$ D. $2+\sqrt{3}$ E. NOTA

18. Given that $\int_a^r (-2x^2-7x+30)dx$ is maximized, find $\left[\frac{|a|}{a}a^2+\frac{|r|}{r}r^2\right]$.

A. -43 B. -30 C. 30 D. 42 E. NOTA

19. Let $f(x)=\dfrac{2x+4}{3x-2}$ and let $g(x)$ be the inverse of $f(x)$. What is $\int_1^3 (f(x)-g(x))\,dx$?

A. $\frac{4}{3}+\frac{16\ln 7}{9}$ B. $\frac{8}{3}+\frac{32\ln 7}{9}$ C. $\frac{16\ln 7+9\ln\frac{625}{81}-15}{9}$ D. DNE E. NOTA

20. Find the area bound by the curve $y=\cosh(x)$ and the lines $y=0, x=\ln 2$, and $x=\ln\frac{1}{2}$. (Hint: $\cosh(x)=\frac{e^x+e^{-x}}{2}$)

A. $\frac{3}{4}$ B. $\frac{5}{4}$ C. $\frac{3}{2}$ D. $\frac{5}{2}$ E. NOTA

21. Let $f(x)=-x^2+2x$, initially. Hurricane Zeta begins to push $f(x)$ upward such that it is moving in the positive y-direction at 2 units/sec. At what rate, in units^2/sec, is the area bound by $f(x)$ and the x-axis changing four seconds after Hurricane Zeta starts?

A. 6 B. 8 C. 12 D. 16 E. NOTA

22. $f(x)=\int_0^{x^2} \sin(\sqrt{t})dt$. Compute $f'(\frac{\pi}{3})$.

A. $\frac{\sqrt{3}}{3}$ B. $\frac{\sqrt{3}}{2}$ C. $\frac{\pi\sqrt{3}}{3}$ D. $\frac{\pi\sqrt{3}}{2}$ E. NOTA

23. Evaluate: $\int_0^{\frac{1}{2}} \sqrt{\frac{1-x}{1+x}}dx$.

A. $\frac{\pi+3\sqrt{3}}{6}$ B. $\frac{\pi+3\sqrt{3}-6}{6}$ C. $\frac{\pi-2}{2}$ D. $\frac{2\pi-3}{6}$ E. NOTA

24. Evaluate: $\int_2^4 \left(\frac{x}{2}\right)^x (\ln(x)-\ln(2)+1)\,dx$

A. 3 B. 6 C. 7.5 D. 15 E. NOTA

25. Let $f'(x) = 3x^2 - 12x + 5$. Given that $f(2) = f(1) + f(0)$, find $f(-2)$.

A. -48 B. -36 C. -24 D. -12 E. NOTA

26. Evaluate: $\int_0^{-\frac{\pi}{3}} \frac{\sin(x)}{\cos(x)+1} dx$.

A. $\ln \frac{3}{4}$ B. $\ln \frac{4}{3}$ C. $\ln 2$ D. $\ln 4$ E. NOTA

27. What is $\int_{-2}^{2} [x] dx$?

A. -2 B. 0 C. 2 D. 4 E. NOTA

28. The number of Hahvahd students that are scared of Handsome Dan is directly proportional to the rate of change of the number of scared Hahvahd students as well as the time, in months, since the start of the year (consider it to be $t = 1$ at present). In two months, how many Hahvahd students will be scared of Handsome Dan, given that currently 1337 students are scared of him and in one month 5348 students will be scared of Handsome Dan?

A. 9359 B. 12033 C. 42784 D. 53905 E. NOTA

29. Evaluate: $\lim_{b\to\infty} \int_{\frac{1}{b}}^{b} \frac{\ln(x)}{x^2+1} dx$.

A. 0 B. $\frac{\pi}{2} \ln 2$ C. $\pi \ln 2$ D. DNE E. NOTA

30. Evaluate: $\int_{-7}^{-11} 2x dy$

A. -72 B. -36 C. 36 D. 72 E. NOTA