*2009 Mu Sequences and Series*

1. Find the sum of the infinite geometric series $\frac{5}{7}+\frac{5}{8}+\frac{35}{64}+...$

A. 5 B. $44/7$ C. 6 D. $40/7$ E. NOTA

2. The sequence $\left\langle \frac{(-1)^n}{n^2} \right\rangle_{n=1}^{\infty}$ is:

A. convergent but not monotone B. unbounded but not monotone C. monotone and convergent D. bounded above but not below E. NOTA

3. Given that the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...+\frac{(-1)^{n+1}}{n}+...$ converges conditionally to $\ln 2$, find the sum of the series

$1+\frac{1}{3}-\frac{1}{2}+\frac{1}{5}+\frac{1}{7}-\frac{1}{4}+...+\frac{1}{4n-3}+\frac{1}{4n-1}-\frac{1}{2n}+...$

A. $\ln 2$ B. $1.5\ln 2$ C. $0.5\ln 2$ D. $2\ln 2$ E. NOTA

4. Which of the following is true about the series $3-3+3-3+3-3+...(-3)^{n+1}+...$?

A. converges absolutely to 0 B. converges conditionally to 3 C. converges conditionally to 1.5 D. diverges E. NOTA

5. Which is/are true about the sequence $\left\langle \frac{(-1)^{n+1}}{n} \right\rangle_{n=1}^{\infty}$?

I. there is an arrangement of the terms to form a series whose sum is $e$
II. there is an arrangement of the terms to form a series whose sum is 1
III. every arrangement forms a series whose sum is $\ln 2$
IV. there is an arrangement of the terms to form a series whose sum is 0

A. I & II only B. III only C. II & IV only D. I, II, & IV only E. NOTA

6. Which of the following is true about $A=\sum_{n=1}^{2009}\frac{1}{n}$ and $B=\ln 2010$? (Hint: use an integral)

A. $A=B$ B. $A<B$ C. $A>B$ D. cannot be determined without a calculator E. NOTA

7. Which test may be used to show the series $\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n}}{n+1}$ converges?

A. ratio test B. alternating series test C. direct comparison test D. limit comparison test E. NOTA

8. Which of the following functions has a Maclaurin series that converges to the function, but not on $(-\infty,\infty)$?

A. $\sin x$ B. $e^x$ C. $\tan^{-1}x$ D. $\cosh x$ E. NOTA

9. An infinite geometric sequence has first term $\frac{4}{9}$. What is the smallest second term of the series if the sum converges to some positive integer?

A. $20/81$ B. $5/9$ C. $28/81$ D. there is no smallest second term E. NOTA

10. The p-series test for convergence of a series is a direct consequence of what other test?

A. ratio test B. integral test C. comparison test D. root test E. NOTA

11. Find the sum: $\sum_{k=1}^{\infty} \frac{k^2+1}{2^k}$

A. 6 B. $91/15$ C. 7 D. $50/7$ E. NOTA

12. Find the sum: $\sum_{k=1}^{\infty} \frac{3k^3-k}{2^k}$

A. $529/7$ B. $1139/15$ C. 72 D. 76 E. NOTA

13. What is the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(x-2)^{n+1}}{(n+1)\cdot 3^{n+1}}$

A. $-1<x<5$ B. $-1\le x<5$ C. $-1<x\le 5$ D. $-1\le x\le 5$ E. NOTA

14. Find the value: $\sqrt{10+\sqrt{670+\sqrt{10+\sqrt{670+\ldots}}}}$

A. 6 B. $6\sqrt{3}$ C. $4\sqrt[4]{6}$ D. $3\sqrt{6}$ E. NOTA

15. Which of the following applies to the series $\sum_{n=1}^{\infty} \frac{1}{2^n-1}$?

A. diverges by ratio test B. converges by alternating series test C. converges by limit comparison test D. diverges by integral test E. NOTA

16. For which positive integers $k$ is the series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(kn)!}$ convergent?

A. $k\ge 1$ B. $k\ge 2$ C. $k\ge 3$ D. $k\ge 4$ E. NOTA

17. Estimate the value of $\int_0^1 e^{-x^2}\,dx$ using a 6th-degree Taylor polynomial for $e^{-x^2}$ centered at $a=0$.

A. $26/35$ B. $18/23$ C. $9/11$ D. $4/7$ E. NOTA

18. The coefficient of the $x^4$ term in the power series representation of $f(x)=\sin^2 x$ is:

A. $-1/3$ B. $8/45$ C. 0 D. 1 E. NOTA

19. For which of the following series is the ratio test inconclusive about the series' convergence or divergence?

A. $\sum_{n=1}^{\infty}\frac{10n+3}{n(2)^n}$ B. $\sum_{n=1}^{\infty}\frac{n}{2n^3+1}$ C. $\sum_{n=1}^{\infty}\frac{n(7)^n}{n!}$ D. $\sum_{n=1}^{\infty}\frac{(-3)^n}{n(4)^n}$ E. NOTA

20. If the $n$th partial sum of the series $\sum_{n=1}^{\infty}a_n$ is $\frac{n-1}{n+1}$, find $a_n$ when $n\geq 2$.

A. $\frac{3}{(n+1)^2}$ B. $\frac{n}{n+4}$ C. $\frac{2}{n(n+1)}$ D. $\frac{4}{(n+1)(n+2)}$ E. NOTA

21. Which is true about the series $\sum_{n=1}^{\infty}\frac{(-1)^{\frac{n(n+1)}{2}}}{3^n}$?

A. it converges conditionally to $-9/20$ B. it converges absolutely to $-9/20$ C. it converges conditionally to $-2/5$ D. it converges absolutely to $-2/5$ E. NOTA

22. Consider two sequences, $\langle a_n\rangle_{n=1}^{\infty}$ and $\langle b_n\rangle_{n=1}^{\infty}$, where $a_n=2^{n-1}$ and $b_n=\frac{n^5-10n^4+55n^3-110n^2+184n}{120}$.

What is the smallest value of $n$ such that $a_n\neq b_n$?

A. 3 B. 5 C. 7 D. $a_n=b_n$ for all $n$ E. NOTA

23. Find the sum: $\sum_{n=0}^{\infty}\frac{1}{2^{n-1}(n!)}$

A. $2\sqrt{e}$ B. $\sqrt{e}$ C. $\sqrt{e}-1$ D. $2\sqrt{e}-1$ E. NOTA

24. Define a sequence recursively by $a_{n+1}=\left(\sqrt[n]{a_n}-\frac{1}{n(n+1)}\right)^{n+1}$ with $a_1=2$. What is the limit of this sequence?

A. 1 B. $e$ C. 2 D. $\pi$ E. NOTA

25. In order for $\sqrt{a+\sqrt{b+\sqrt{a+\sqrt{b+\ldots}}}}$ to converge, what must be true about positive integers $a$ and $b$?

A. $a>b$ B. $a^2>b$ C. $b>a$ D. $b>a^2$ E. NOTA

26. What is the Taylor series for the function $f(x)=\ln x-\ln 2$ centered at $a=2$?

A. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-2)^n}{n(2)^n}$ B. $\sum_{n=2}^{\infty}\frac{(-1)^{n+1}(x-2)^n}{n(2)^n}$ C. $\sum_{n=1}^{\infty}\frac{(-1)^n(x-2)^n}{n(2)^n}$ D. $\sum_{n=2}^{\infty}\frac{(-1)^n(x-2)^n}{n(2)^n}$

E. NOTA

27. Which of the following applies to the series $\sum_{n=1}^{\infty}\frac{1}{n\ln n}$?

A. converges to $2^{\ln 2}$ B. converges to $e^{0.5}$ C. converges to $e^{\ln 2}$ D. diverges E. NOTA

28. Evaluate $\int\frac{\tan^{-1}x}{x}dx$ as a power series.

A. $C+\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)^2}$ B. $C+\sum_{n=0}^{\infty}\frac{(-1)^{n+1} x^{2n+1}}{(2n+1)^2}$ C. $C+\sum_{n=0}^{\infty}\frac{(-1)^{2n+1} x^{2n+1}}{(2n+1)^2}$ D. $C+\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n-1)^2}$

E. NOTA

29. Find the sum of the series $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10}+\frac{1}{12}+\frac{1}{14}+\ldots$, where each term is the reciprocal of an integer whose only factors are 2, 3, 5, or 7.

A. the series diverges B. $\frac{27}{8}$ C. $\frac{31}{8}$ D. $\frac{35}{8}$ E. NOTA

30. Which is true about $A=\sum_{k=0}^{\infty}\left(\frac{(-1)^k}{(2k)!}\left(\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\right)^{2k}\right)$ and $B=\sum_{k=0}^{\infty}\left(\frac{(-1)^k}{(2k+1)!}\left(\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\right)^{2k+1}\right)$?

A. $B>A$ B. $A>B$ C. $A=B$ D. unable to be determined E. NOTA

Free Response Tiebreakers

TB1. What is the common name of the sequence given by $a_n=\frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n-\left(\frac{1-\sqrt{5}}{2}\right)^n\right)$?

TB2. Find the sum: $\sum_{m=1}^{2009}\left(\sum_{n=1}^{2009}\left(\frac{2^n}{2^m+2^n}\right)\right)$

TB3. Find the value of $\sqrt{8+\frac{8}{\sqrt{8+\frac{8}{\sqrt{8+\frac{8}{\ldots}}}}}}$.